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PREDICTION OF FORMING LIMIT CURVES OF SHEET METALS USING HILL'S 1993 USER-FRIENDLY YIELD CRITERION OF ANISOTROPIC MATERIALS

SIGUANG XU and KLAUS J. WEINMANN

Department of Mechanical Engineering and Engineering Mechanics, Michigan Technological University, Houghton, MI 49931, U.S.A.

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Abstract—A user-friendly yield criterion was proposed by Hill in 1993, which utilizes five independent material parameters in representing the yield locus. In the present investigation, an attempt is made to analyze forming limits in sheet metals based on this yield criterion and the M–K approach. Comparison of the predicted results with experimental data indicates that Hill's 1993 yield criterion is able to characterize the localized necking of both aluminum and AK steel. A parametric study is carried out to investigate the influence of material parameters (r_0, r_{90}, σ_0, σ_{90} and σ_b) on forming limits, which shows that the shape of the yield locus has a significant influence on limit strains.

Keywords: limit curves, sheet metals, yield criterion.

NOTATION

a, b	subscript for uniform region and groove
c, p, q	non-dimensional parameters in Hill's yield function
r_0, r_{45}, r_{90}	ratios of transverse to through thickness strains under uniaxial tension at 0°, 45° and 90° to the rolling direction
r	ratio of transverse to through thickness strains of planar isotropic materials
t_0, t	initial and current thickness of the sheet
A, B, C	disposable parameters
K, n, m	constant, strain hardening exponent, rate sensitivity exponent in constitutive equation
M	exponent in Hill's 1979 yield criterion
f_0	factor for initial non-homogeneity
α_0	ratio of yield stresses of uniaxial tension at 0° and 90° to the rolling direction (σ_0/σ_{90})
α_b	ratio of uniaxial tension yield stress in 0° direction to biaxial tension yield stress (σ_0/σ_b)
σ_b, σ_u	yield stresses under biaxial tension and uniaxial tension
σ_0, σ_{90}	yield stresses under uniaxial tension at 0° and 90° to the rolling direction
$d\varepsilon_1$, $d\varepsilon_2$, $d\varepsilon_3$	increments of strain components
$\dot{\varepsilon}_1$, $\dot{\varepsilon}_2$	components of strain rate
σ_e, $\dot{\bar{\varepsilon}}$, $\bar{\varepsilon}$	effective stress, effective strain rate and effective strain
σ_1, σ_2	stress components
α, ρ	ratio of strain increments ($d\varepsilon_2/d\varepsilon_1$) and ratio of stresses (σ_2/σ_1)
ε_0, Δ	initial strain and finite difference symbol

INTRODUCTION

Forming limit diagrams (FLDs) are affected by the shapes of yield surfaces. Since a method of calculating the FLDs was proposed by Marciniak and Kuczynski [1] and Marciniak *et al.* [2], much work has been focused on forming limit analysis using Hill's 1948 quadratic yield criterion [3]. Reasonable correlation between experimental data and this theory have been found for aluminum-killed steel with r-values between 1 and 2 [4]. The analyses with this criterion show that the position of forming limit curves depends on the initial non-homogeneity, strain hardening exponent n, and rate sensitive exponent m. The shape of forming limit curves for the stretching operation, on the other hand, is affected by the r-value and somewhat by m. With an increase of r-value, the forming limit strain is predicted to decrease significantly [5–7]. Experimentally, however, the dependence of limit strain on the r-value is not observed to be as pronounced as the theory predicts [2, 8].

From Hill's 1948 theory, the relationship between the yield stress under balanced biaxial tension and the yield stress under uniaxial tension, $\sigma_b = \sqrt{(r+1)/2}\sigma_u$, may be obtained. According to this relationship, the yield stress under biaxial tension should be less than the yield stress under uniaxial tension for r-values less than unity (such as aluminum). However, experimental data [9] show that the biaxial tension yield stress of some aluminum is higher than the uniaxial tension yield stress, a phenomenon known as "anomalous behavior" of aluminum.

In 1979, Hill proposed a second criterion of anisotropic plasticity and gave four simplified forms for planar isotropic sheet metals [10]. All the four forms are able to describe the anomalous behavior. Using this criterion, Parmar and Mellor [7] predicted much lower forming limits for r-values less than that with Hill's 1948 criterion. Lian *et al.* [11] investigated the range of the applicable stress exponent M in the four simplified forms of this criterion. They pointed out that the yield locus of the fourth form remains convex as long as M is larger than unity while the other three forms have a narrower range for M. By comparing with experimental data, Kobayashi *et al.* [12] concluded that the stress exponent M varies with accumulated strain. If a fixed value of M is used, it may cause a discrepancy between the predicted yield locus and the experimental one.

Since the shape of the yield surface has a significant influence on the predicted forming limit strains for stretching operations and sheet metals may have different yield surfaces due to their physical characteristics [13], numerous anisotropic yield criteria have been postulated. For example, Hosford [14] proposed a yield function based on the upper bound Bishop and Hill analysis [15,16]. Although the form of the Hosford yield function may coincide with Hill's 1979 yield function under a special condition [10], the stress exponent M in Hosford's yield function is independent of r-value and has much higher values (of 6 and 8 for bcc and fcc metals, respectively). Based on this criterion, Graf and Hosford [17,18] have shown that the r-value has much less an effect on predicted FLDs, a result which shows agreement with experimental observation. To accommodate the anomalous behavior of planar isotropic sheets, Bassani advocated a yield function with four independent parameters [19] and Gotoh proposed a fourth-degree polynomial yield function [20]. Based on Logan and Hosford's yield function [21], Barlat postulated a yield function that involves shear stress, which seems to be able to represent some characteristics of aluminum sheets [22, 23]. It may be shown that in most of the above-mentioned criteria there exists a fixed relationship among the r-value, the yield stress under uniaxial tension and the yield stress under biaxial tension (as we have shown for Hill's 1948 criterion). This restricts their application to some materials.

For some materials such as copper, the ratio r_0/r_{90} may be far from unity, while σ_0/σ_{90} may or may not be equal to one. To deal with this type of sheet metal property, Hill proposed a user-friendly yield function in 1993 [24]. In this criterion, there are five independent material parameters, thus offering flexibility in representing the yield locus. This yield function has been used by Hill *et al.* to model the plastic work contours of 70–30 brass in polar coordinates of stress. The function is shown to agree well with experimental data [25]. The purpose of the present investigation is focused on the application of this newly proposed criterion to forming limit analysis using the M–K method. A parametric study is carried out to investigate the influence of the material parameters on forming limits. Predictions based on the criteria proposed by Hill in 1993, 1979 and 1948 are compared with experimental data and discussed

HILL'S USER-FRIENDLY YIELD CRITERION

The general form of the criterion proposed by Hill in 1993 [24] is

$$\frac{\sigma_1^2}{\sigma_0^2} - \frac{c\sigma_1\sigma_2}{\sigma_0\sigma_{90}} + \frac{\sigma_2^2}{\sigma_{90}^2} + \left\{(p+q) - \frac{p\sigma_1 + q\sigma_2}{\sigma_b}\right\}\frac{\sigma_1\sigma_2}{\sigma_0\sigma_{90}} = 1, \tag{1}$$

where c, p and q are non-dimensional parameters and given by

$$\frac{c}{\sigma_0\sigma_{90}} = \frac{1}{\sigma_0^2} + \frac{1}{\sigma_{90}^2} - \frac{1}{\sigma_b^2}, \tag{2}$$

$$\begin{aligned}\left(\frac{1}{\sigma_0} + \frac{1}{\sigma_{90}} - \frac{1}{\sigma_b}\right)p &= \frac{2r_0(\sigma_b - \sigma_{90})}{(1+r_0)\sigma_0^2} - \frac{2r_{90}\sigma_b}{(1+r_{90})\sigma_{90}^2} + \frac{c}{\sigma_0}\\ \left(\frac{1}{\sigma_0} + \frac{1}{\sigma_{90}} - \frac{1}{\sigma_b}\right)q &= \frac{2r_{90}(\sigma_b - \sigma_0)}{(1+r_{90})\sigma_{90}^2} - \frac{2r_0\sigma_b}{(1+r_0)\sigma_0^2} + \frac{c}{\sigma_{90}}.\end{aligned} \tag{3}$$

In the above equations, σ_b is the yield stress under balanced biaxial tension, σ_0 and σ_{90} are yield stresses under uniaxial tension at 0° and 90° to the rolling direction, respectively, and r_0 and r_{90} are ratios of transverse to through-thickness increments of logarithmic strain under σ_0 and σ_{90}, respectively. It is noted that there are five independent material parameters σ_0, σ_{90}, σ_b, r_0, and r_{90} among which no presumed relationship is postulated. These parameters need be determined for each material experimentally.

If for a given material the yield stresses at 0° and 90° to the rolling direction are the same, then Eqn (1) will take the form

$$\sigma_1^2 - \left(2 - \frac{\sigma_u^2}{\sigma_b^2}\right)\sigma_1\sigma_2 + \sigma_2^2 + \left\{(p+q) - \frac{p\sigma_1 + q\sigma_2}{\sigma_b}\right\}\sigma_1\sigma_2 = \sigma_u^2, \tag{4}$$

where σ_u is the yield stress under uniaxial tension, and p and q are given by

$$\begin{aligned}\left(2 - \frac{\sigma_u}{\sigma_b}\right)p &= \frac{2}{1+r_0} + \left(\frac{2r_0}{1+r_0} - \frac{2r_{90}}{1+r_{90}}\right)\frac{\sigma_b}{\sigma_u} - \frac{\sigma_u^2}{\sigma_b^2},\\ \left(2 - \frac{\sigma_u}{\sigma_b}\right)q &= \frac{2}{1+r_{90}} + \left(\frac{2r_{90}}{1+r_{90}} - \frac{2r_0}{1+r_0}\right)\frac{\sigma_b}{\sigma_u} - \frac{\sigma_u^2}{\sigma_b^2},\end{aligned} \tag{5}$$

or equivalently

$$\begin{aligned}\frac{1}{2}(p+q) &= \left(\frac{r_0}{1+r_0} + \frac{r_{90}}{1+r_{90}} - \frac{\sigma_u^2}{\sigma_b^2}\right)\Big/\left(2 - \frac{\sigma_u}{\sigma_b}\right)\\ \frac{1}{2}(p-q) &= \left(\frac{r_0}{1+r_0} - \frac{r_{90}}{1+r_{90}}\right)\Big/\left(2 - \frac{\sigma_u}{\sigma_b}\right).\end{aligned} \tag{6}$$

Equations (5) and (6) are different from Eqns (18) and (19) in Hill's original paper in which r_0 and r_{90} in the numerators are missing.

If two parameters α_0 and α_b are introduced so that $\sigma_0 = \alpha_0\sigma_{90}$ and $\sigma_0 = \alpha_b\sigma_b$, then Eqns (1)–(3) may be written as

$$\frac{\sigma_1^2}{\sigma_0^2} - \alpha_0\frac{c\sigma_1\sigma_2}{\sigma_0^2} + \alpha_0^2\frac{\sigma_2^2}{\sigma_0^2} + \alpha_0\left\{(p+q) - \alpha_b\frac{p\sigma_1 + q\sigma_2}{\sigma_0}\right\}\frac{\sigma_1\sigma_2}{\sigma_0^2} = 1, \tag{7}$$

where

$$c = \frac{1}{\alpha_0} + \alpha_0 - \frac{\alpha_b^2}{\alpha_0} \tag{8}$$

and

$$\begin{aligned}(1+\alpha_0-\alpha_b)p &= \frac{2r_0(\alpha_0-\alpha_b)}{(1+r_0)\alpha_0\alpha_b} - \frac{2r_{90}\alpha_0^2}{(1+r_{90})\alpha_b} + c,\\ (1+\alpha_0-\alpha_b)q &= \frac{2r_{90}(1-\alpha_b)\alpha_0^2}{(1+r_{90})\alpha_b} - \frac{2r_0}{(1+r_0)\alpha_b} + \alpha_0 c.\end{aligned} \tag{9}$$

Equation (7) allows for the investigation of the influence of parameters α_0, α_b, r_0 and r_{90} on the yield locus in a co-ordinate system of normalized stresses σ_1/σ_0 and σ_2/σ_0. For planar isotropic materials, the effect of r-value on the yield locus is shown in Fig. 1 where α_b and α_0 remain unity. With an increase of r-value, the yield locus becomes more rounded. Figure 2 shows the influence of r_{90} on the yield locus while other parameters remain constant. A higher value of r_{90} shifts the locus in the 90° direction. Figure 3 demonstrates the effect of α_b on the yield locus. It is observed that a decrease in α_b leads to an increase in the yield stress under biaxial tension. The yield locus moves outward in the 45° direction. Figure 4 shows the effect of α_0 on the yield locus. For a clear illustration of the influence of α_0 on the yield locus and for the sake of comparison with predicted FLDs, α_b in Figure 4 is set to be 0.9. Since for a fixed σ_0 a decrease in α_0 indicates an increase in σ_{90}, the yield locus is pushed outward in the 90° direction.

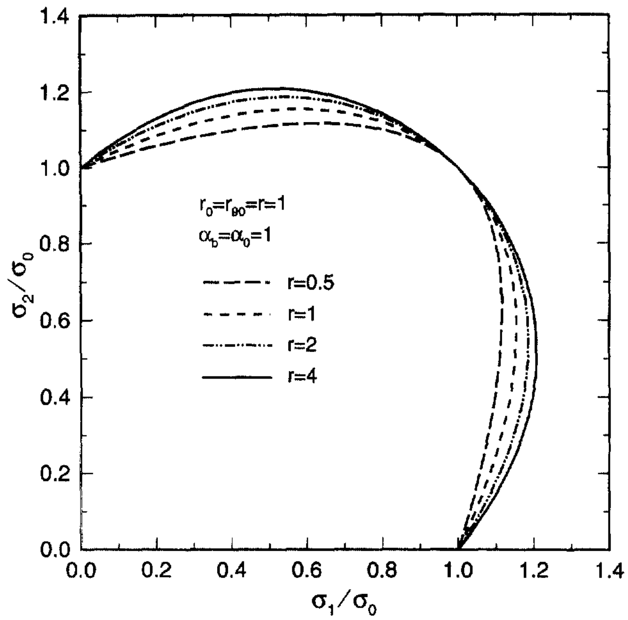

Fig. 1. Influence of planar isotropic r-value on yield locus.

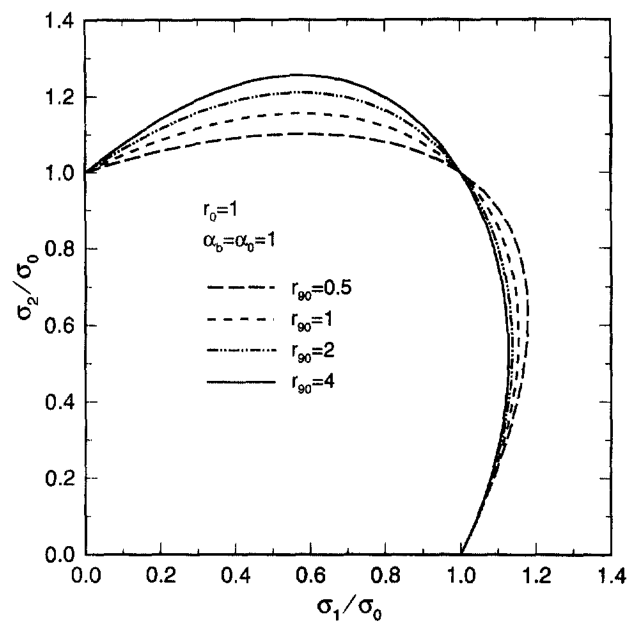

Fig. 2. Influence of anisotropic strain ratio r_{90} on yield locus.

APPLICATION OF THE HILL CRITERION TO FORMING LIMIT ANALYSIS

Since this criterion only involves two principal stresses which are acting along in-plane orthotropic directions, it is applicable to analysis of the right-hand side of forming limit diagram. To analyze the left-hand side of FLDs, the yield criterion needs to include shear stress. Otherwise, assumptions on the direction of principal stresses inside the inhomogeneity of the sheet would have to be made. In the present work, only the right-hand side of the FLD will be discussed.

Under the condition of biaxial tension, the groove will be perpendicular to the σ_1 direction as shown in Fig. 5. The equilibrium and deformation compatibility conditions between the groove and

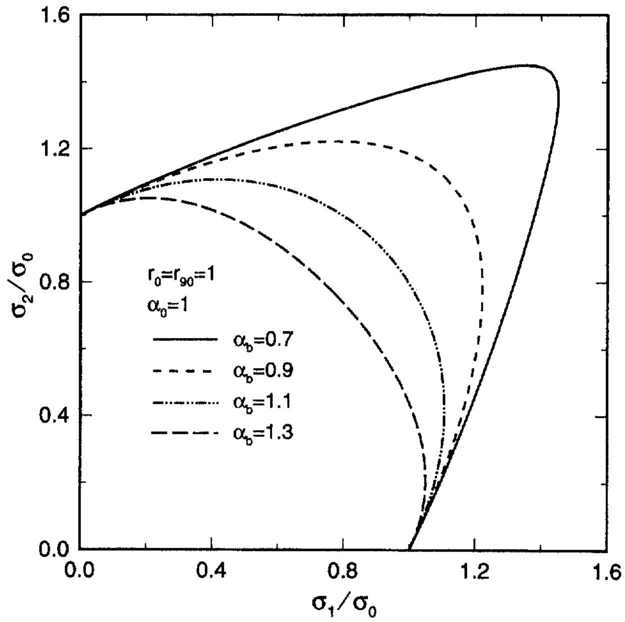

Fig. 3. Effect of parameter α_b on yield locus.

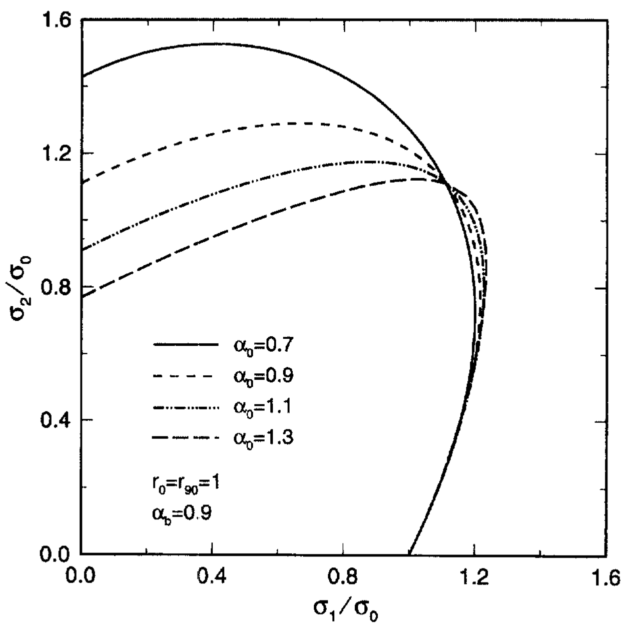

Fig. 4. Effect of parameter α_0 on yield locus.

the outside region are

$$\sigma_{1a}t_a = \sigma_{1b}t_b, \tag{10}$$

$$d\varepsilon_{2a} = d\varepsilon_{2b}, \tag{11}$$

where subscript "a" and "b" denote the uniform region and the groove, respectively. From the definition of logarithmic strain, the thickness of the sheet can be expressed as

$$t = t_0 \exp(\varepsilon_3). \tag{12}$$

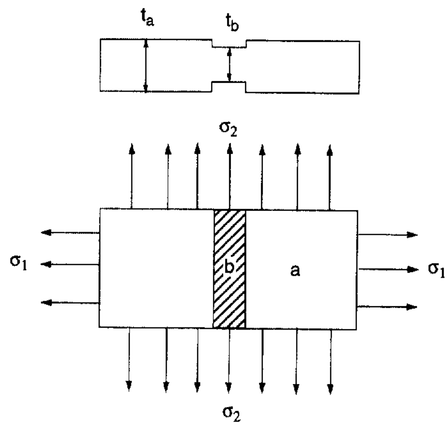

Fig. 5. Schematic of the groove.

Combining Eqn (10) with Eqn (12), the equilibrium equation becomes

$$\sigma_{1a}\exp(\varepsilon_{3a}) = f_0\sigma_{1b}\exp(\varepsilon_{3b}), \tag{13}$$

where $f_0 = t_{0b}/t_{0a}$ is the initial non-homogeneity factor. The effective stress and effective strain rate are defined as

$$\begin{aligned}\sigma_e\dot{\bar{\varepsilon}} &= \sigma_1\dot{\varepsilon}_1 + \sigma_2\dot{\varepsilon}_2 \\ &= \sigma_1\dot{\varepsilon}_1(1 + \alpha\rho),\end{aligned} \tag{14}$$

where $\alpha = \sigma_2/\sigma_1$ is the stress ratio, $\rho = \dot{\varepsilon}_2/\dot{\varepsilon}_1$ is the strain rate ratio, and σ_0 is chosen as effective stress, that is $\sigma_0 = \sigma_e$.

From the flow rule, the strain rate ratio is obtained as

$$\rho = \frac{\dot{\varepsilon}_2}{\dot{\varepsilon}_1} = \frac{2\alpha_0^2\sigma_2 - \alpha_0 c\sigma_1 + \alpha_0((p+q) - \alpha_b((p\sigma_1 + 2q\sigma_2)/\sigma_0))\sigma_1}{2\sigma_1 - \alpha_0 c\sigma_2 + \alpha_0((p+q) - \alpha_b((2p\sigma_1 + q\sigma_2)/\sigma_0))\sigma_2}. \tag{15}$$

From the definitions in Eqn (14) and volume constancy, strain increments may be given by

$$\mathrm{d}\varepsilon_1 = \frac{\sigma_0\,\mathrm{d}\bar{\varepsilon}}{\sigma_1(1 + \alpha\rho)}, \tag{16}$$

$$\mathrm{d}\varepsilon_2 = \rho\,\mathrm{d}\varepsilon_1, \tag{17}$$

$$\mathrm{d}\varepsilon_3 = -\,\mathrm{d}\varepsilon_1(1 + \rho). \tag{18}$$

To facilitate the analysis, isotropic strain hardening is assumed and the constitutive equation is selected in the form of the power law

$$\sigma_e = K(\varepsilon_0 + \bar{\varepsilon})^n\dot{\bar{\varepsilon}}^m. \tag{19}$$

By rearranging the yield function (Eqn (7)), the following equation for normalized stress σ_1/σ_0:

$$A\left(\frac{\sigma_1}{\sigma_0}\right)^3 + B\left(\frac{\sigma_1}{\sigma_0}\right)^2 + 1 = 0 \tag{20}$$

is obtained, where

$$\begin{aligned}A &= \alpha_0\alpha_b\alpha(p + q\alpha), \\ B &= -(1 + \alpha_0^2\alpha^2 + (p + q - c)\alpha\alpha_0).\end{aligned} \tag{21}$$

From Eqn (15), the normalized stress may also be expressed as

$$C = \frac{\sigma_1}{\sigma_0} = \frac{2(\rho - \alpha\alpha_0^2) + (p + q - c)(\rho\alpha - 1)\alpha_0}{((2p + q\alpha)\alpha\rho - (p + 2q\alpha))\alpha_b\alpha_0}. \tag{22}$$

Combining Eqn (20) with Eqn (22) finally results in the governing equation for stress ratio and strain rate ratio:

$$AC^3 + BC^2 + 1 = 0. \tag{23}$$

Equation (23) involves the two variables α and ρ. Once strain rate ratio ρ is given, stress ratio α can be solved for, and then C is determined using Eqn (22). For the right-hand side of the FLD, the value of α is between α_l and α_r, with the two limiting values corresponding to plane strain and balanced biaxial tension. The range of ρ is normally within 0 and 1. However, Eqn (23) is a highly non-linear equation. There may exist more than one solution for a given value of ρ. In order to find the correct solution, the following steps are used:

(1) Solve Eqn (23) and find all solutions of α between α_l and α_r.
(2) Eliminate solutions corresponding to negative C.
(3) For solutions with positive C, ρ is subjected to a small positive perturbation $\Delta\rho$. Solve Eqn (23) again and compare

$$\left.\frac{\sigma_1}{\sigma_0}\right|_{\rho+\Delta\rho} \quad \text{with} \left.\frac{\sigma_1}{\sigma_0}\right|_{\rho} \cdot \text{If} \left.\frac{\sigma_1}{\sigma_0}\right|_{\rho+\Delta\rho} < \left.\frac{\sigma_1}{\sigma_0}\right|_{\rho},$$

then α is the correct solution for the given value of ρ.

Since σ_1/σ_0 is determined from Eqn (23), Eqn (13) may be written as

$$\frac{(\sigma_1/\sigma_0)_a}{(\sigma_1/\sigma_0)_b} = f_0 \exp(\varepsilon_{3b} - \varepsilon_{3a}) \left(\frac{\varepsilon_0 + \bar{\varepsilon}_b}{\varepsilon_0 + \bar{\varepsilon}_a}\right)^n \left(\frac{\dot{\bar{\varepsilon}}_b}{\dot{\bar{\varepsilon}}_a}\right)^m. \tag{24}$$

The above equation is the equilibrium equation that needs to be solved under the condition of Eqns (4) and (23). For this purpose, a numerical procedure similar to that used by Graf and Hosford [18] is employed. Equation (24) is thus changed to the form

$$\exp(\varepsilon_{3a})(\sigma_1/\sigma_0)_a(\bar{\varepsilon}_a + \Delta\bar{\varepsilon}_a)^n(\Delta\bar{\varepsilon}_a)^m = f_0 \exp(\varepsilon_{3b})(\sigma_1/\sigma_0)_b(\bar{\varepsilon}_b + \Delta\bar{\varepsilon}_b)^n(\Delta\bar{\varepsilon}_b)^m. \tag{25}$$

Equations (4), (23) and (25) constitute a non-linear system of equations. To solve these equations simultaneously, the following procedure was used:

(1) Impose a strain increment $\Delta\varepsilon_{1b}$ inside the groove.
(2) Assume a strain rate ratio ρ_a outside the groove, and use Eqns (22) and (23) to determine C_a and α_a.
(3) Select a strain increment $\Delta\varepsilon_{1a}$, calculate the left-hand side of Eqn (25) and use Eqn (4) to calculate the strain rate ratio ρ_b in the groove.
(4) Solve Eqn (23) to obtain α_a, and calculate the right-hand side of Eqn (25).
(5) Check if the left-hand side is equal to the right-hand side in Eqn (25).
(6) Adjust $\Delta\varepsilon_{1a}$ and repeat steps (3) through (5) until both sides of Eqn (25) match.
(7) Update total strains

$$\bar{\varepsilon} = \bar{\varepsilon} + \Delta\bar{\varepsilon},$$

$$\varepsilon_3 = \varepsilon_3 + \Delta\varepsilon_3,$$

where

$$\Delta\bar{\varepsilon} = (1 + \alpha\rho)C\Delta\varepsilon_1,$$

$$\Delta\varepsilon_3 = -\Delta\varepsilon_1(1 + \rho).$$

(8) Impose another strain increment $\Delta\varepsilon_{1b}$, repeat steps (3)–(7) until $\Delta\varepsilon_{1a}/\Delta\varepsilon_{1b}$ approaches a small number which was selected to be 0.15 in the calculation since numerical tests indicated that a number less than this value has little effect on the accuracy of calculated results.

RESULTS AND DISCUSSION

A comprehensive investigation into the forming limits of aluminum alloy 6111-T4 has been conducted by Graf and Hosford [26]. Their experimental data served as a basis for comparison with the theory presented in this paper. The comparison is shown in Fig. 6, where average values of experimental data for n, r_{90}, r_0 and α_0 were used in the calculation. Since under the condition of near-biaxial tension ridging affects failure, and necks occur parallel to the rolling direction [26], the initial non-homogeneity factor f_0 for the groove parallel to the rolling direction (RD) is chosen to be less than that for the groove parallel to the transverse direction (TD). Experimental data [26] show a negative rate sensitivity exponent ($-0.003 \sim -0.004$) for Aluminum 6111-T4. To facilitate analysis, however, the rate sensitivity exponent is chosen as zero in the calculation. By assuming that the as-received material has not been prestrained, the initial strain in Eqn (19) is neglected in the present analysis. Crystal plasticity analyses indicate that the minimum value of α_b is 0.847 [27]. Thus, 0.9 was deemed representative for α_b in the analysis due to lack of relevant experimental data. It is shown in Fig. 6 that the predicted FLDs (solid lines) give agreement with experimental data, where the dots represent strains measured on circles for which necking was noted, while the open circles represent strains measured on neck-free circles [26]. It is observed that the predicted forming limit strains follow the experimental data closely for the case when the major strain is parallel to the transverse direction. However, it can be shown that if a value lower than 0.9 is used for α_b, the predicted forming limit strains will coincide with experimental data more closely for the case with the major strain parallel to RD.

The dash and long-dash lines in Fig. 6 represent the limit strains predicted using Hill's 1993 criterion for the case of planar isotropy. The r-value is assumed to be the same as r_0 and r_{90} for σ_1 parallel and perpendicular to the rolling direction, respectively. Predictions from Hill's 1948 yield criterion and the fourth form of Hill's 1979 yield criterion are also shown in the same figure (where, the stress exponent M in Hill's 1979 criterion is determined from the r-value and α_b). Comparison among the predictions from the three yield criteria shows that Hill's 1993 criterion explains the trend of experimental data for aluminum 6111-T4 best. Near biaxial tension limit strains predicted from Hill's 1948 and 1979 criteria are observed to give over- and under-estimates, respectively. It is noted that other yield criteria, such as Hosford's high exponent yield criterion [18], may also give reasonable predictions of forming limits for aluminum alloys. However, comparison of Hill's criteria with criteria proposed by other researchers is beyond the scope of the present investigation.

Figure 7 compares the predicted limit strains with experimental data for AK steel [28]. Published experimental results for AK steel differ considerably [5] due to the variations in sheet thickness and material properties. Hecker's data seem to provide higher limit strains than those of other published data [5]. In the calculations, the values for n, α_0, r_0, r_{90}, and r are from Hecker's tests [28]. The rate

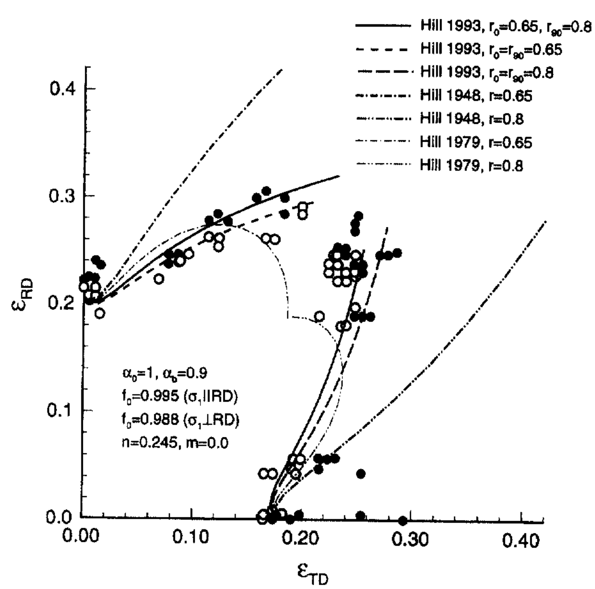

Fig. 6. Comparison of predicted FLDs with experimental data of aluminum 6111-T4 sheet. The dots and circles represent measured strains on neck-affected and neck-free strains circles [26]. Lines are the predicted results.

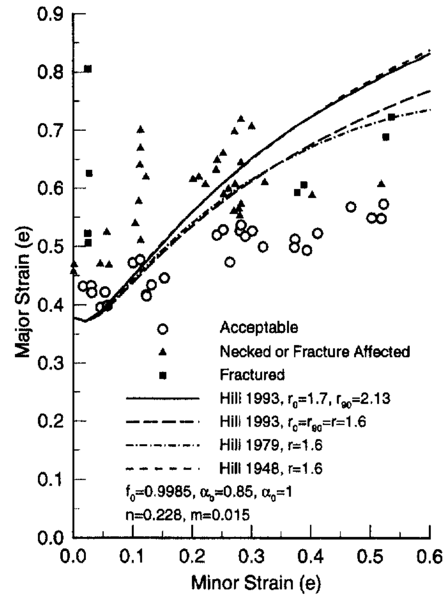

Fig. 7. Comparison of predicted limit strains with experimental data for AK steel. The circles, triangles and rectangles represent experimental data [28]. Lines are predicted results.

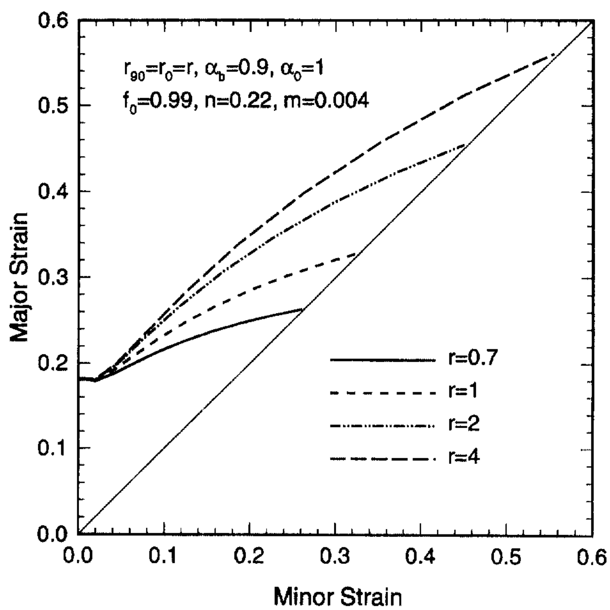

Fig. 8. Effect of planar anisotropic r-value on forming limit strains.

sensitivity exponent m (0.015) is selected within the range of experimental data [29]. Since Hill's 1948 quadratic criterion is generally applicable for AK steel, experimental data as well as this criterion serve as a basis for determining the value of α_b as 0.85 [30]. It is observed that for Hill's 1993 yield criterion, limit strains predicted with an average r-value of 1.6 are lower than those predicted by assuming that the material is in-plane anisotropic. This is due to the fact that r_{45} is not involved in Hill's 1993 yield criterion when in-plane anisotropy is considered. Also plotted in the figure are the limit strains predicted from Hill's 1948 yield criterion and the fourth form of Hill's 1979 yield criterion. It is observed that for AK steel, the difference among the predictions from these three criteria are insignificant if compared with the predictions for aluminum in Fig. 6. In fact, if the value of α_b is given by a critical value of $\sqrt{2/(r+1)}$ (which is 0.877 with r equal to 1.6), Hill's 1979 and 1993

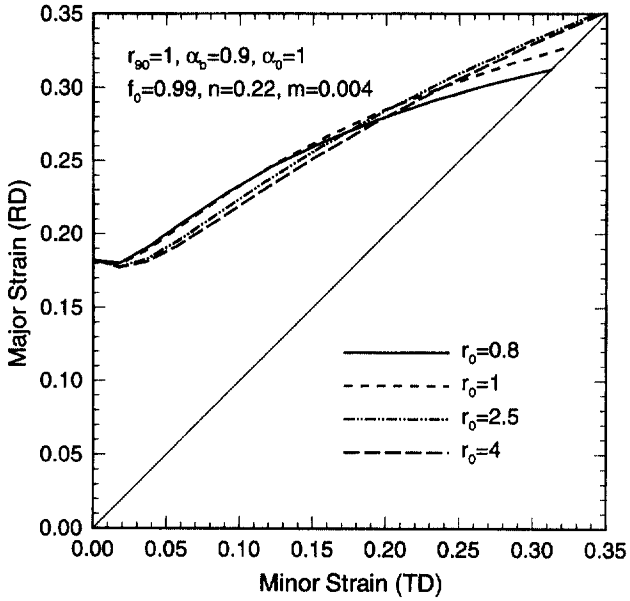

Fig. 9. Influence of anisotropic strain ratio r_0 on forming limit strains.

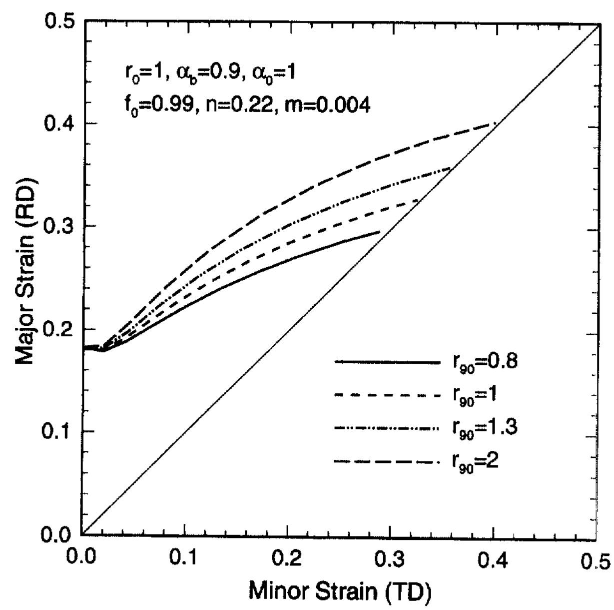

Fig. 10. Influence of anisotropic strain ratio r_{90} on forming limit strains.

yield functions will reduce to Hill's 1948 yield function. Since $\alpha_b = 0.85$ is close to this critical value, the forming limit predictions from these three criteria should be expected to be similar as well.

To investigate the effect of material parameters on forming limits, the initial non-homogeneity parameter f_0, the n-value and the m-value are arbitrarily chosen as 0.99, 0.22 and 0.004, respectively in the subsequent analysis. The low value of f_0 causes the calculated limit strain for the plane strain condition to be lower than the n-value. Figure 8 shows the effect of the planar isotropic r-value on the FLD. Due to the fact that the r-value is an independent parameter in Hill's 1993 yield criterion, its value exerts a significant influence on the shape of the yield locus (Fig. 1). As a result, the forming limit strain is sensitive to the r-value although experimental data may not show the evidence for such a r-value dependence. It is also observed that variations of the forming limit with the r-value

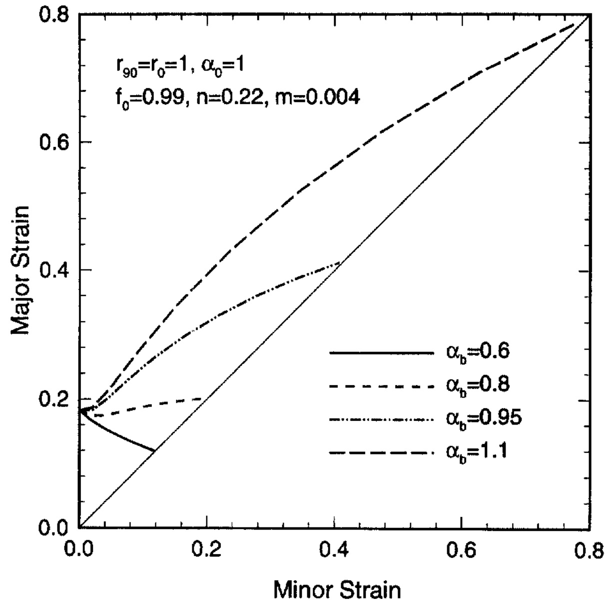

Fig. 11. Effect of parameter α_b on forming limit strains.

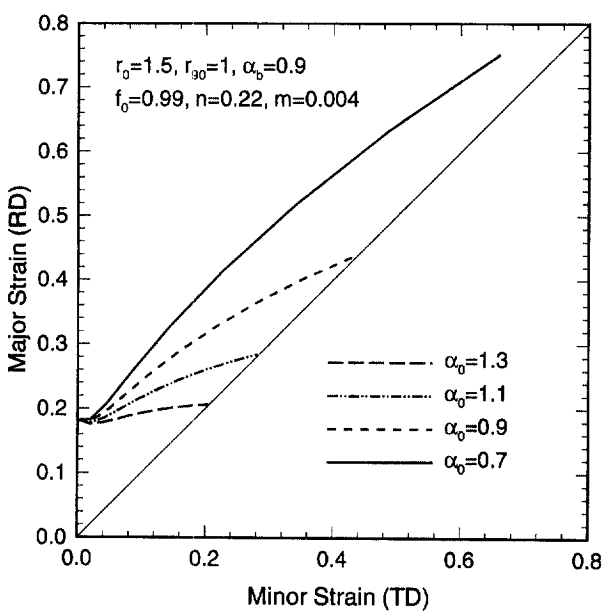

Fig. 12. Effect of parameter α_0 on forming limit strains.

contradicts the prediction by Hill's 1948 yield criterion [7]. However, if an increase in the r-value occurs simultaneously with a decrease in α_b (such as AK steel), the predicted forming limit would be expected to decrease and follow the experimental data (Fig. 7). Figures 9 and 10 illustrate the influence of r_0 and r_{90} on forming limit strains, where the orientation of the groove is perpendicular to the rolling direction. It is interesting to note that r_0 influences the FLD less than r_{90}. However, with increasing r_{90}, stretchability improves significantly in the 0° direction.

Figure 11 illustrates the influence of the parameter α_b on the FLD. An increase in α_b is equivalent to a decrease in σ_b in comparison with σ_0 (Fig. 3). This leads to a more rounded yield locus and higher forming limits. Figure 12 shows that an increase in α_0 results in a decrease in forming limits.

Barlat [13] argues that the sharpness of the yield locus at the point of biaxial tension is a factor controlling limiting strains. He used the ratio of yield stress under plane strain to yield stress under biaxial tension ($P = \sigma_p/\sigma_b$) to characterize the shape of the yield locus. Comparison of Figs 3 and 4 with Figs 11 and 12 indicates that the present analysis generally agrees with Barlat's statement that decreasing sharpness of the yield locus in the biaxial tension direction will promote larger limiting strains. However, for Hill's user-friendly yield criterion the parameter P proposed by Barlat does not seem suitable for describing the sharpness of the yield locus in the biaxial tension direction since σ_b is an independent parameter, see Fig. 4.

CONCLUSIONS

Forming limits in stretching operations are analyzed based on Hill's 1993 user-friendly yield criterion. A non-linear governing equation is derived to establish the relationship between strain rate ratio and stress ratio under the condition of proportional loading. To find the correct solution to this equation, a numerical perturbation technique is pursued, which is based on the characteristic of stretching operations where the principal stress σ_1 decreases when the deformation mode switches from plane strain to balanced biaxial tension. Comparison of predicted forming limit strains with experimental data shows that Hill's 1993 user-friendly criterion is applicable to both steel and aluminum. Published experiments [31] revealed that the shape of yield loci is dependent on the deformation history. In the present investigation, however, the stress ratios α_0 and α_b are kept constant during the deformation process, a situation where the yield locus expands in scale only but not in shape. This may contribute to discrepancies between experimental data and predictions. Analysis shows that the shape of the yield locus has a significant influence on limiting strains. A flattened shape of the yield locus in biaxial tension permits high limiting strains in stretching.

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