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Product-Oriented Sensitivity Analysis for Multistation Compliant Assemblies

Dimensional variation in assembled products directly affects product performance. To reduce dimensional variation, it is necessary that an assembly be robust. A robust assembly is less sensitive to input variation from the product and process components, such as incoming parts, subassemblies, fixtures, and welding guns. In order to effectively understand the sensitivity of an assembly to input variation, an appropriate set of metrics must be defined. In this paper, three product-oriented indices, including pattern sensitivity index, component sensitivity index, and station sensitivity index, are defined. These indices can be utilized to measure the variation influence of a pattern, an individual part, and/or component, and components at a particular station to the dimensional quality of a final assembly. Additionally, the relationships among these sensitivity indices are established. Based on these relationships, the ranges of the sensitivity indices are derived. Finally, a case study of a sheet metal assembly is presented and discussed to illustrate the applicability of these metrics. [DOI: 10.1115/1.2735341]

Keywords: sensitivity analysis, compliant assembly, variation propagation

1 Introduction

Multistation compliant assembly process is one of the most widely used in the automotive, airplane, furniture, and home appliance manufacturing. The dimensional quality of assembled products plays an important role in cost, designed functionality, and customer satisfactions of the final assembly. In order to obtain high-quality assemblies, several approaches and strategies have been studied. In general, these approaches fall into two categories: source variation reduction and sensitivity analysis (robust design). High quality in assemblies can be obviously achieved through reducing variation directly at the source. However, it was also noted that reducing the variation at the source becomes increasingly complex, time consuming, and costly as the variation diminishes. Because of that, the second approach, the sensitivity analysis in dimensional variation, has been developed [1–18]. High-quality products can be achieved by reducing the sensitivity of the assembly dimensional variation to the source of variation instead of directly reducing the variation at the source. The importance of this method has been emphasized in manufacturing processes since it was proposed by Taguchi and Wu in the 1970s [1]. After that, the sensitivity analysis has been increasingly gaining attention in more manufacturing areas.

In multistation compliant assembly processes, the impact of an incoming part or component on the product dimensional accuracy depends on its variation, its geometry, its material properties, its fixture layout, and other process information. The evaluation of this impact through sensitivity analysis is very important for designing a robust product and process. For example, sensitivity analysis demonstrates the importance of each part and, therefore, the most important part can be focused to improve the dimensional accuracy of final product. An effective sensitivity analysis requires a set of indices. The challenges to definition of the indices for multistage compliant assemblies come from: (i) a set of indices instead of only index are needed to evaluate the impact of a part from different aspects and (ii) the set of indices should be related to each other.

Several studies have been conducted to establish effective sensitivity indices from different aspects. Ting and Long [9] conducted a sensitivity analysis for mechanisms based on a linear relationship between inputs and outputs. A boundary for the sensitivity was derived from the properties of the Rayleigh's quotient. In addition, they presented two guidelines to minimize the variation transmission. Ceglarek and Shi [19] proposed a product joint evaluation index and a critical part determination index to evaluate how the different joints affect the robustness of a design. The proposed indices, which are based on the direct interactions between the components, can be used as an analytical tool to analyze and benchmark different designs regarding their dimensional integrity. Gao et al. [11] defined tolerance sensitivity as the influence of individual component tolerances on the variation of a critical assembly feature or dimension. They proposed a new method for determining tolerance sensitivity using vector loop assembly tolerance models and evaluated the derivative matrix of the equations with respect to the assembly variables. This derivative matrix was used to calculate the sensitivity matrix. Ding et al. [14] stated that sensitivity analysis is more effective as an evaluation tool at the design stage and that this is due to its input-independent property. Additionally, a process oriented sensitivity analysis at the system level based on a state-space equation of variation propagation for multistation rigid assembly systems was performed. The sensitivity indices were defined for fixtures, stations, and whole systems. These sensitivity indices, however, are applicable to *rigid* assembly processes only. In addition, the relationship among these indices was not derived. Hu et al. [15] proposed a method for evaluating the robustness of compliant assembly systems based on a variation simulation model. They defined variation transmission ratios and sensitivity indices, and analyzed the range of a predefined sensitivity index. The method, however, is applicable for *single* stations only. In summary, the sensitivity analysis in manufacturing systems falls into two categories: product oriented and process oriented. Product-oriented sensitivity analysis characterizes the impact of part variation on final product quality. In comparison, process-oriented sensitivity analysis measures the impact of process variation, such as fixture variation, on final product quality [14]. Some selected studies for sensitivity analysis are presented in Table 1 for a comparison.

The purpose of this paper is to present a set of product-oriented

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	Station level	System level
Process oriented	Cai et al. [10], Wang [20]	Ding et al. [14]
Product oriented	Gao et al. [11], Hu et al. [15], Ceglarek and Shi [19]	To be developed in this paper

sensitivity indices for the analysis of multistation compliant assemblies. Three indices are defined: the pattern sensitivity index, the component sensitivity index, and the station sensitivity index. The relationship among these indices and the indices' ranges are also derived. The remainder of the paper is organized as follows: Section 2 reviews the background in multistation simulation model for compliant assemblies used to construct the proposed sensitivity indices. Section 3 introduces the three proposed product-oriented sensitivity indices and the methodology used to define them. Section 4 discusses a case study that illustrates how to apply the proposed indices in the different aspects of product design. Finally, conclusions are drawn in Sec. 5.

2   Background on Multistation Compliant Assembly Modeling

To describe the dimensional relationship between an assembly and its components at the station level, Liu et al. [21] and Liu and Hu [22] proposed a linear model

$$V_w = S \cdot V_u \tag{1}$$

where V_w and V_u are vectors that represent the dimensional variation of the key product characteristics (KPCs) of the assembly and its components, respectively; and S is the sensitivity matrix, which can be obtained by the influence coefficient method presented in Liu and Hu [22]. Based on this method, the deviations of measurement points due to a unit deviation of each welding point are calculated and recorded as one column of the sensitivity matrix.

To describe the dimensional variation propagation along the stations for a compliant assembly process, Camelio et al. [23] proposed to use a state-space model

$$X_i = A_i X_{i-1} + B_i U_i + W_i$$

$$Y_i = C_i X_i + E_i \tag{2}$$

where X_i and X_{i-1} are the state vectors; A_i is the state matrix; B_i is the input matrix; U_i is the input vector; C_i is the observation matrix; W_i is the disturbance vector; and E_i is the measurement noise vector.

The state equation in the state-space model (Eq. (2)) for the dimensional variation propagation for compliant assemblies can be rewritten as follows:

$$X_i = (S_i - D_i + I)[X_{i-1} + M_i(X_{i-1} - U_i^{3-2-1})]$$
$$- (S_i - D_i)(U_i^{n-3} + U_i^g) + W_i \tag{3}$$

where the state vector X_i is defined as a vector of dimensional variation, including the KPCs points and key control characteristics (KCCs) points for all the components at the ith station.

In order to obtain A_i and B_i, the relocating matrix M and deformation matrix D were defined and derived [23]. The relocating matrix explains how the state vector changes due to the change on the locating scheme from the previous station to the current station. On the other hand, the deformation matrix considers the initial shape of the parts or subassemblies.

In addition, U_i is defined as the input vector, which includes the dimensional variation of the n-2-1 locating and holding fixtures and the welding guns. The input vector U_i can be decomposed into locating fixtures, which are denoted as U_i^{3-2-1}; the n-3 ($n > 3$) additional holding fixtures, denoted as U_i^{n-3}, and the dimen-

sional variation of the assembly tools, which is denoted as U_i^g. In a compliant assembly system, the assembly tools variation is usually corresponding to the welding gun variation.

With an assumption that the fixture scheme is 3-2-1 rather than n-2-1 ($n > 3$) and welding guns are perfect in this paper, U_i^{n-3} and U_i^g are correspondingly equal to zero, and; therefore, Eq. (3) can be simplified as follows:

$$X_i = (S_i - D_i + I)[X_{i-1} + M_i(X_{i-1} - U_i^{3-2-1})] + W_i$$
$$= (S_i - D_i + I)(I + M_i)X_{i-1} - (S_i - D_i + I)M_iU_i^{3-2-1} + W_i$$
$$= A_iX_{i-1} + B_iU_i^{3-2-1} + W_i \tag{4}$$

where

$$A_i = (S_i - D_i + I)(I + M_i)$$

$$B_i = -(S_i - D_i + I)M_i$$

Without the assumption that the fixture scheme is 3-2-1 and welding guns are prefect, Eq. (3) can still be rewritten as a state-space form with more complex expression of state matrix A and input matrix B.

Considering the sequential assembly process, the state equation and observation equation can be written as follows:

$$X_i = \Phi(i,1)X_0 + \sum_{j=1}^{i} [\Psi(i,j)U_j]$$

$$Y_i = C_i\Phi(i,1)X_0 + C_i\sum_{j=1}^{i} [\Psi(i,j)U_j] \tag{5}$$

where

$$\Phi(i,j) = A_iA_{i-1}, \ldots, A_{j+1}A_j \quad (i \geq j) \quad \text{and} \quad \Phi(j,j) = A_j$$

$$\Psi(i,j) = A_iA_{i-1}, \ldots, A_{j+1}B_j \quad (i > j) \quad \text{and} \quad \Psi(j,j) = B_j$$

X_0 is the deviation vector for the source points of all the incoming parts.

The noise and disturbance effects are neglected in Eq. (5) for convenience. The equation describes how the deviation of each part or subassembly propagates during the assembly process and is accumulated into the final assembly. In order to obtain the equations for the variance propagation, it is assumed that the fixtures variances are independent of the part variances. Under this assumption, the following equation about the variances can be rewritten from Eq. (5):

$$\Sigma_Y = \gamma(1)\Sigma_{X_0}\gamma^T(1) + \sum_{j=1}^{N} [\Gamma(j)\Sigma_{U_j}\Gamma^T(j)] \tag{6}$$

where

$$\gamma(t) = C\Phi(N,t)$$

$$\Gamma(t) = C\Psi(N,t)$$

and Σ_Y is the covariance matrix of measurement points on the final assembly, N is the number of station in the assembly system, Σ_{U_i} is the covariance matrix of the errors in the 3-2-1 fixtures at the ith station, and Σ_{X_0} is the covariance matrix for the source points on all the incoming parts.

Equation (6) shows that the variance of final assemblies comes from two main sources: the variance of the processes (fixtures) and the variance of incoming parts. Correspondingly, there are two types of sensitivity analysis. The first type of analysis refers to the sensitivity of the final assembly dimensional variation to the dimensional variation of fixtures. This type of sensitivity analysis is called process-oriented sensitivity analysis. The second type is about the sensitivity of the final assembly dimensional variation to its components dimensional variation, which is called product-

oriented sensitivity analysis. The process-oriented sensitivity analysis for compliant assemblies can be conducted by applying the sensitivity analysis methodologies for the fixtures in a multistation rigid assembly system ([14]). Process-oriented sensitivity is not covered in that paper. In compliant assembly systems, component dimensional variation will alter the location of the locating fixture points, clamp points, welding points, and measurement points on the components. In contrast, in a rigid assembly system, component dimensional variation only affects the final product variation through impacting the fixture location points and measurement points. Therefore, component variation will affect product variation more significantly in compliant assembly systems than in rigid assembly systems. Product-oriented sensitivity analysis plays an important role in the design and analysis of compliant assembly systems. In addition, for the fixtures at a particular station, it can be reasonably assumed that the variation of the fixtures is independent of each other in the process-oriented sensitivity analysis. In product-oriented sensitivity analysis for a component in compliant assembly systems, the variation of the source points on the same surface of the component is obviously dependent on each other. Therefore, the process-oriented sensitivity analysis methodologies proposed by Ding et al. [14] cannot directly be applied into the product-oriented sensitivity analysis for multistation compliant assemblies. The remainder of this paper focuses on the methodologies of product-oriented sensitivity analysis for multistation compliant assemblies.

Product-oriented sensitivity analyzes how the dimensional variation of final products is sensitive to the source variation of parts or components. Since it is assumed that the tooling variation is independent of the variation of parts, the tooling variation will not have any contribution to the product-oriented sensitivity. Therefore, Eq. (6) can be rewritten as follows by ignoring the tooling variation contribution in the assembly product:

$$\Sigma_Y = \gamma(1)\Sigma_{X_0}\gamma^T(1) \tag{7}$$

In Eq. (7), Σ_{X_0} can be written as a block diagonal matrix by assuming that the source variation between parts is independent of each other

$$\Sigma_{X_0} = \begin{bmatrix} \Sigma_{X_{p\_1}} & 0 & \cdots & 0 \\ 0 & \Sigma_{X_{p\_2}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Sigma_{X_{p\_k}} \end{bmatrix}$$

where $\Sigma_{X_{p\_i}}$ is the covariance matrix for the source points on the ith part. The dimension of matrix $\Sigma_{X_{p\_i}}$ is $m_i \times m_i$, where m_i is the number of source points in the ith part, and k is the number of incoming parts in the system. Correspondingly, the matrix $\gamma(1)$ can be partitioned as $\gamma(1) = (\gamma(1)_1, \gamma(1)_2, \ldots, \gamma(1)_k)$. Then, Eq. (7) can be rewritten as

$$\Sigma_Y = \gamma(1)\Sigma_{X_0}\gamma^T(1) = \gamma(1)_1\Sigma_{X_{p\_1}}\gamma^T(1)_1 + \gamma(1)_2\Sigma_{X_{p\_2}}\gamma^T(1)_2$$
$$+ \cdots + \gamma(1)_k\Sigma_{X_{p\_k}}\gamma^T(1)_k = \sum_{i=1}^{k}[\gamma(1)_i\Sigma_{X_{p\_i}}\gamma^T(1)_i] \tag{8}$$

3 Product-Oriented Sensitivity Analysis

A typical assembly process is illustrated in Fig. 1, parts P1 and P2 are assembled at station 1 forming the subassembly A1. Then, subassembly A1 is assembled with the part P3 at station 2 becoming subassembly A2, which is then assembled with part P4 at station 3 to create the final assembly A3. At the end of the process, the final assembly is measured at the station 4. Based on the product-oriented sensitivity analysis for this system, the sensitivity of the final assembly A3 to the variance of parts or subassemblies P1, P2, P3, P4, A1, or A2 is called component sensitivity. In addition, the sensitivity of the variance of the final assembly A3 to
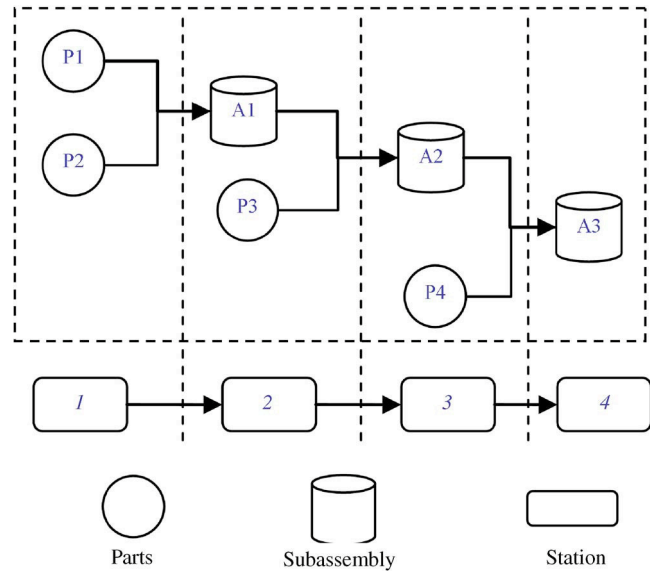

Fig. 1 Assembly process example

the variance of A1 and P3, which are the components at the second station, is called station sensitivity. Additionally, pattern sensitivity quantifies how the variance of final assemblies is sensitive to the variation patterns of a component.

3.1 Pattern Sensitivity. A pattern is a correlated movement of multiple measurement points due to their geometric relationships and certain root causes in a manufacturing system. A pattern can be extracted from the correlation matrices of measurement points. It can also be designated based on the effects of a certain root cause on the derivation of measurement points. The importance of applying the patterns on the variation analysis is that they will often have physical interpretations of root causes for the processes or products. Since a pattern is very important in manufacturing system diagnosis and product quality improvement, several studies have been done in extracting patterns, interpreting the information in terms of root causes, and establishing guidelines for quality improvement [24–27].

Pattern sensitivity studies the robustness of the assembly dimensional variation to the variation patterns of a part or subassembly. This sensitivity metric is more advantageous than a point-based sensitivity to identify variation root causes. The traditional point-based sensitivity is defined as the sensitivity of product dimensional variation to the variation of each source point on a part. The pattern sensitivity analysis takes the covariance information of source points into account, whereas the point-based sensitivity inappropriately assumes that the source points are independent even if the points are on the same surface of a part.

In order to conduct the pattern sensitivity for a part, for example, part i, the other parts are assumed to be perfect. Therefore, the variation propagation on Eq. (8) can be written as

$$\Sigma_Y = \gamma(1)_1\Sigma_{X_{p\_1}}\gamma^T(1)_1 + \gamma(1)_2\Sigma_{X_{p\_2}}\gamma^T(1)_2 + \cdots$$
$$+ \gamma(1)_k\Sigma_{X_{p\_k}}\gamma^T(1)_k = \gamma(1)_i\Sigma_{X_{p\_i}}\gamma^T(1)_i \tag{9}$$

The covariance matrix of the ith part $\Sigma_{X_{p\_i}}$ can be decomposed into variation patterns using principal component analysis [24]. In this paper, pattern obtained using principal component analysis is used to illustrate the pattern sensitivity index though patterns extracted from other methods can also be applied

$$\Sigma_{X_{p\_i}} = \boldsymbol{P}\Lambda\boldsymbol{P}^T = \begin{pmatrix} \boldsymbol{P}_{i1} \\ \boldsymbol{P}_{i2} \\ \vdots \\ \boldsymbol{P}_{im_i} \end{pmatrix} \begin{pmatrix} \lambda_{i1} & 0 & \cdots & 0 \\ 0 & \lambda_{i2} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \lambda_{im_i} \end{pmatrix} \begin{pmatrix} \boldsymbol{P}_{i1} \\ \boldsymbol{P}_{i2} \\ \vdots \\ \boldsymbol{P}_{im_i} \end{pmatrix}^T$$

$$= \lambda_{i1}\boldsymbol{P}_{i1}\boldsymbol{P}_{i1}^T + \lambda_{i2}\boldsymbol{P}_{i2}\boldsymbol{P}_{i2}^T + \cdots + \lambda_{im_i}\boldsymbol{P}_{im_i}\boldsymbol{P}_{im_i}^T = \sum_{j=1}^{m_i} (\lambda_{ij}\boldsymbol{P}_{ij}\boldsymbol{P}_{ij}^T)$$

(10)

Substituting Eq. (10) into Eq. (9),

$$\Sigma_Y = \gamma(1)\Sigma_{X_{p\_i}}\gamma^T(1)_i = \lambda_{i1}\gamma(1)_i\boldsymbol{P}_{i1}\boldsymbol{P}_{i1}^T\gamma^T(1)_i + \lambda_{i2}\gamma(1)_i\boldsymbol{P}_{i2}\boldsymbol{P}_{i2}^T\gamma^T(1)_i$$

$$+ \cdots + \lambda_{im_i}\gamma(1)_i\boldsymbol{P}_{im_i}\boldsymbol{P}_{im_i}^T\gamma^T(1)_i = \sum_{j=1}^{m_i} [\lambda_{ij}\gamma(1)_i\boldsymbol{P}_{ij}\boldsymbol{P}_{ij}^T\gamma^T(1)_i]$$

Therefore,

$$\sum_{i=1}^{d} \sigma_{Y_i}^2 = \text{Tr}(\Sigma_Y) = \sum_{j=1}^{m_i} \{\lambda_{ij} \text{Tr}[\gamma(1)_i\boldsymbol{P}_{ij}\boldsymbol{P}_{ij}^T\gamma^T(1)_i]\} \qquad (11)$$

where $\text{Tr}(\cdot)$ is trace operator for a matrix and d is the number of measurement points on final assembly.

Based on Eq. (11) and assuming that the eigenvalue λ_{ij} is equal to one and the other eigenvalues are equal to zeros, the sensitivity of the jth variation pattern of the ith part can be defined as follows:

$$S_{prn\_ij} = \text{Tr}[\gamma(1)_i\boldsymbol{P}_{ij}\boldsymbol{P}_{ij}^T\gamma^T(1)_i] \qquad (12)$$

This definition measures the sum of the variance of measurement points on the final product induced by one unit variance of a particular pattern. Through this definition, the impact of most significant variation patterns of a part on the product dimensional variation can be quantified. Therefore, the importance of the root causes related to these patterns can be measured and the guidelines for product designers and process engineers to improve the quality of the product can be established.

From Eq. (12), it can be derived that

$$S_{prn\_ij} = \text{Tr}[\gamma(1)_i\boldsymbol{P}_{ij}\boldsymbol{P}_{ij}^T\gamma^T(1)_i] = \|\gamma(1)_i\boldsymbol{P}_{ij}\|_2 = \frac{\langle\gamma(1)_i\boldsymbol{P}_{ij}, \gamma(1)_i\boldsymbol{P}_{ij}\rangle}{\langle\boldsymbol{P}_{ij},\boldsymbol{P}_{ij}\rangle}$$

$$= \frac{\langle\gamma^T(1)_i\gamma(1)_i\boldsymbol{P}_{ij}, \boldsymbol{P}_{ij}\rangle}{\langle\boldsymbol{P}_{ij},\boldsymbol{P}_{ij}\rangle}$$

where $\|\ \|_2$ is the operator for the Euclidean norm of vectors, angular brackets are the operator for the inner product of a pair of vectors.

Because the matrix $\gamma^T(1)_i\gamma(1)_i$ is a real symmetric matrix, an equation can be derived based on the property of the Rayleigh's quotient as follows:

$$\eta_r^2 = \varsigma_r \leq S_{prn\_ij} \leq \varsigma_1 = \eta_1^2 \qquad (13)$$

where ς_r and ς_1 are minimum and maximum eigenvalues of the matrix $\gamma^T(1)_i\gamma(1)_i$, respectively. η_r and η_1 are the corresponding minimum and maximum singular values of the matrix $\gamma(1)_i$.

Equation (12) shows that the sensitivities of all the patterns of part i are related to the same matrix $\gamma(1)_i$. In addition, from Eq. (13), it can be seen that the range of the sensitivity of a pattern only depends on the singular values of the matrix $\gamma(1)_i$. Therefore, all the patterns of a part, for example, part i, have the same range for their sensitivity indices since the patterns have the same matrix $\gamma(1)_i$, which is independent of the covariance matrix of the incoming parts.

3.2 Component Sensitivity. Component sensitivity studies the sensitivity of the assembly dimensional variation to the variation of one of the parts/components of the assembly. The impact of each component on the final product dimensional variation can

be quantified and compared through this metric. It will be also shown that the component sensitivity is the weighted sum of all the sensitivities of the component patterns. Based on this relationship and the range for the pattern sensitivity of a component, the range for the sensitivity of the component is derived in this section.

As Eq. (8) shown, all the parts of an assembly contribute to the assembly variation. In order to measure the sensitivity of the assembly variation to the variation of an individual part, for example, part i, it is assumed that all the other parts are perfect. Then,

$$\Sigma_Y = \sum_{i=1}^{k} [\gamma(1)\Sigma_{X_{p\_i}}\gamma^T(1)_i] = \gamma(1)_i\Sigma_{X_{p\_i}}\gamma^T(1)_i \qquad (14)$$

Equation (14) is the same as Eq. (9). Based on Eq. (14), the sensitivity of an individual part, for example, the ith part, can be formulized as follows:

$$S_{prt\_i} = \frac{\text{Tr}(\Sigma_Y)}{\text{Tr}(\Sigma_{X_{p\_i}})} = \frac{\text{Tr}[\gamma(1)_i\Sigma_{X_{p\_i}}\gamma^T(1)_i]}{\text{Tr}(\Sigma_{X_{p\_i}})} \qquad (15)$$

where the trace of covariance matrix of the measurement points on the final product Σ_Y represents the sum of the variance of the measurement points. The trace of the covariance matrix $\Sigma_{X_{p\_i}}$ represents the sum of the variance of the source points on the ith part. This definition quantifies the joint impact of the variance of all the source points on the ith part to the final product variance.

From this definition, it can be derived that

$$S_{prt\_i} = \frac{\text{Tr}[\gamma(1)_i\Sigma_{X_{p\_i}}\gamma^T(1)_i]}{\text{Tr}(\Sigma_{X_{p\_i}})} = \frac{\sum_{j=1}^{m_i} \{\text{Tr}[\lambda_{ij}\gamma(1)_i\boldsymbol{P}_{ij}\boldsymbol{P}_{ij}^T\gamma^T(1)_i]\}}{\text{Tr}(\Sigma_{X_{p\_i}})}$$

$$= \sum_{j=1}^{m_i} \left\{ \frac{\lambda_{ij}}{\text{Tr}(\Sigma_{X_{p\_i}})} \text{Tr}[\gamma(1)_i\boldsymbol{P}_{ij}\boldsymbol{P}_{ij}^T\gamma^T(1)_i] \right\}$$

$$= \sum_{j=1}^{m_i} \left[\frac{\lambda_{ij}}{\text{Tr}(\Sigma_{X_{p\_i}})} S_{prn\_ij} \right]$$

where λ_{ij} is the jth eigenvalue of the covariance matrix $\Sigma_{X_{p\_i}}$ of the source points on the ith part and it is also the variance of the pattern. S_{prn_ij} is the sensitivity index for the jth variation pattern of the ith part.

The above equation shows that the sensitivity of a part is equal to the weighed sum of the sensitivities of all the patterns of the part. And the weight coefficients are

$$\text{Coeff}_j = \frac{\lambda_{ij}}{\text{Tr}(\Sigma_{X_{p\_i}})}, \quad \text{where} \quad j = 1, \ldots, m_i$$

Based on the eigenvalues properties of a matrix, it is known that

$$\text{Tr}(\Sigma_{X_{p\_i}}) = \sum_{j=1}^{m_i} \lambda_{ij}$$

Therefore, the sum of all the weight coefficients is equal to one.

$$\sum_{j=1}^{m_i} \text{Coeff}_j = \sum_{j=1}^{m_i} \left[\frac{\lambda_{ij}}{\text{Tr}(\Sigma_{X_{p\_i}})} \right] = 1$$

The sensitivity of a variation pattern of a part has a range as follows:

$$\eta_r^2 \leq S_{prn\_ij} \leq \eta_1^2$$

Based on this equation and the weighted sum relationship between the sensitivities of the patterns of a part and the sensitivity of the part, it can be shown that

$$S_{prt\_i} = \sum_{j=1}^{m_i} \left[\frac{\lambda_{ij}}{\text{Tr}(\Sigma_{X_{p\_i}})} S_{prn\_ij} \right] \le \sum_{j=1}^{m_i} \left[\frac{\lambda_{ij}}{\text{Tr}(\Sigma_{X_{p\_i}})} \eta_1^2 \right] = \eta_1^2$$

and

$$S_{prt\_i} = \sum_{j=1}^{m_i} \left[\frac{\lambda_{ij}}{\text{Tr}(\Sigma_{X_{p\_i}})} S_{prn\_ij} \right] \ge \sum_{j=1}^{m_i} \left[\frac{\lambda_{ij}}{\text{Tr}(\Sigma_{X_{p\_i}})} \eta_r^2 \right] = \eta_r^2$$

Therefore,

$$\eta_r^2 \le S_{prt\_i} \le \eta_1^2 \tag{16}$$

Again, η_r and η_1 are the corresponding minimum and maximum singular values of the matrix $\gamma(1)_i$.

It must be noted that this range is independent of the covariance matrix of incoming parts. This property is important because the component sensitivity can be estimated even when the covariance matrix is unknown, which is not uncommon, especially at the design stage.

3.3 Station Sensitivity. The station sensitivity analysis studies the sensitivity of the assembly variation to the variation of all the parts/components interacting in a particular assembly station. Although at an assembly station, the variation of not only the components and/or parts, but also the fixtures and assembly tooling can contribute to the variation of final assemblies, this sensitivity index quantifies the overall impact of the components assembled at a station to the final product dimensional variation. In addition, the relationship between the sensitivity of a station and the sensitivities of the components at this station is shown in this section. Based on this relationship, the range of the sensitivity for a station is derived.

Similarly to the derivation of Eq. (6), an equation is derived as follows:

$$\Sigma_Y = \gamma(i)\Sigma_{X_{i-1}}\gamma^T(i) + \sum_{j=i}^{N} \left[\Gamma(j)\Sigma_{U_j}\Gamma^T(j) \right] \tag{17}$$

where $\Gamma(j)$ and $\gamma(i)$ were defined in Eq. (6).

This equation describes how the variance of all the fixtures and components from station i to the last station of the system propagate to the final assembly. In contrast, Eq. (6) describes how the variance of all the fixtures and components at all the stations of the system propagate to the final assembly.

Based on the assumption that the variation of fixtures is independent of the variation of parts, the variation of fixtures is neglected for the study of sensitivity analysis for a particular station. Therefore, Eq. (17) is written as follows:

$$\Sigma_Y = \gamma(i)\Sigma_{X_{i-1}}\gamma^T(i) \tag{18}$$

The variance relationship between the final assembly Y and the components after the ith station inclusively, X_{i-1}, is shown in Eq. (18).

The covariance matrix $\Sigma_{X_{i-1}}$ can be written as a block diagonal matrix as follows:

$$\Sigma_{X_{i-}} = \begin{bmatrix} \Sigma_{X_{c\_i1}} & 0 & \cdots & 0 & 0 & \cdots & \cdots & 0 & 0 & \cdots \\ 0 & \Sigma_{X_{c\_i2}} & \cdots & 0 & 0 & \cdots & \cdots & 0 & 0 & \cdots \\ & & \ddots & \vdots & \vdots & \vdots & \vdots & & & \vdots \\ & & & \Sigma_{X_{c\_(i+1)1}} & 0 & \cdots & \cdots & 0 & 0 & \cdots \\ & & & & \Sigma_{X_{c\_(i+1)2}} & \cdots & \cdots & 0 & 0 & \cdots \\ & & & & & \ddots & \vdots & \vdots & \vdots & \vdots \\ & & & & & & \ddots & \vdots & \vdots & \vdots \\ & & & & & & & \Sigma_{X_{c\_N1}} & 0 & \cdots \\ & & & & & & & & \Sigma_{X_{c\_N2}} & \cdots \\ & & & & & & & & & \ddots \end{bmatrix}$$

where $\Sigma_{X_{c\_ij}}$ is the covariance matrix for the source points on the jth component at the ith station. Correspondingly, the matrix $\gamma(i)$ can be partitioned as

$$\gamma(i) = \begin{bmatrix} \gamma(i)_{c\_i1} & \gamma(i)_{c\_i2} & \cdots & \gamma(i)_{c\_(i+1)1} & \gamma(i)_{c\_(i+1)2} & \cdots & \gamma(i)_{c\_N1} & \gamma(i)_{c\_N2} & \cdots \end{bmatrix}$$

Then, Eq. (18) can be written as follows with the assumption that the components on the other stations except station i are perfect:

$$\Sigma_Y = \gamma(i)\Sigma_{X_{i-1}}\gamma^T(i) = \sum_{j=i}^{N} \left\{ \sum_{r=1} \left[\gamma(i)_{c\_jr}\Sigma_{X_{c\_ir}}\gamma^T(i)_{c\_jr} \right] \right\}$$

$$= \sum_{r=1} \left[\gamma(i)_{c\_ir}\Sigma_{X_{c\_ir}}\gamma^T(i)_{c\_ir} \right]$$

Based on this equation, the product-oriented sensitivity for station i is defined as follows:

$$S_{stn\_i} = \frac{\text{Tr}(\Sigma_Y)}{\text{Tr}(\Sigma_{X_{i-1}})} = \frac{\text{Tr}\left\{ \sum_{r=1} \left[\gamma(i)_{c\_ir}\Sigma_{X_{c\_ir}}\gamma^T(i)_{c\_ir} \right] \right\}}{\text{Tr}\left[\sum_{r=1} (\Sigma_{X_{c\_ir}}) \right]} \tag{19}$$

This definition for the sensitivity index shows how the variation of the components at a station will be accumulated on the final assembly. For example, if S_{stn_i} is small, it can be concluded that the component variation at the ith station is diminished on the final assembly. Or else, it is increased. From Eq. (19), it can be derived that

$$S_{srn\_i} \frac{\text{Tr}\left\{ \sum_{r=1} \left[\gamma(i)_{c\_ir} \Sigma_{X_{c\_ir}} \gamma^T(i)_{c\_ir} \right] \right\}}{\text{Tr}\left[\sum_{r=1} (\Sigma_{X_{c\_ir}}) \right]}$$

$$= \sum_{r=1} \left\{ \frac{\text{Tr}[\gamma(i)_{c\_ir} \Sigma_{X_{c\_ir}} \gamma^T(i)_{c\_ir}]}{\sum_{r=1} [\text{Tr}(\Sigma_{X_{c\_ir}})]} \right\}$$

$$= \sum_{r=1} \left\{ \frac{\text{Tr}(\Sigma_{X_{c\_ir}})}{\sum_{r=1} [\text{Tr}(\Sigma_{X_{c\_ir}})]} \frac{\text{Tr}[\gamma(i)_{c\_ir} \Sigma_{X_{c\_ir}} \gamma^T(i)_{c\_ir}]}{\text{Tr}(\Sigma_{X_{c\_ir}})} \right\}$$

$$= \sum_{r=1} \left\{ \frac{\text{Tr}(\Sigma_{X_{c\_ir}})}{\sum_{r=1} [\text{Tr}(\Sigma_{X_{c\_ir}})]} S_{prt\_c\_ir} \right\}$$

where $S_{prt\_c\_ir}$ is the sensitivity index for the rth component at the ith station.

The preceding equation shows that the sensitivity of a station is equal to the weighted sum of the sensitivities of all the components at the station. One of the weight coefficients is

$$\text{Coeff}_r = \frac{\text{Tr}(\Sigma_{X_{c\_ir}})}{\sum_{r=1} [\text{Tr}(\Sigma_{X_{c\_ir}})]}$$

It can also be easily shown that the sum of all the weight coefficients is equal to one. Therefore, the range of the sensitivity for a station is obtained as follows:

$$S_{prt\_c\_ir}^{\min} \leq S_{stn\_i} \leq S_{prt\_c\_ir}^{\max}$$

where $S_{prt\_c\_ir}^{\min}$ and $S_{prt\_c\_ir}^{\max}$ are the minimum and maximum among all the sensitivities for the components at the ith station.

3.4 Summary of Sensitivity Indices. In summary, pattern, component, and station sensitivity index are defined for multistation compliant assemblies from a product point of view. The relationships among these sensitivity metrics are also developed. Based on these relationships, the ranges for these sensitivities are derived that are independent of the covariance matrices of incoming parts and/or components.

Equations (13) and (16), show that the pattern sensitivities and the component sensitivity for the ith part are within the range of η_r^2 and η_1^2, the minimum and maximum singular values of the matrix $\gamma(1)_i$. Therefore, if $\eta_r^2 > 1$, all the pattern sensitivities and component sensitivity will be >1. In other words, variance of the incoming patterns or the variance of the components will be amplified during the assembly process. Similarly, it can be concluded that the variance of the patterns or the variance of the component will be reduced if $\eta_1^2 < 1$.

In addition, the sensitivity indices for all the patterns of a particular component have the same boundary $[\eta_r^2 \quad \eta_1^2]$. An index can be defined as follows:

$$\text{index}_{prn} = \frac{\eta_1^2 - \eta_r^2}{\eta_r^2}$$

Through this index, the sensitivity of the product variance to the different patterns of a particular component can be quantified. For example, if this index is small, all the patterns will have very similar potential impacts on the final product variance. The extreme case is that the unit variance of different patterns has the same impact on the final product variance if this index is equal to zero. In contrast, if this index is big, the patterns will have very different potential impacts on the final product variance. Therefore, this index can be used to evaluate the product robustness to the variances induced by different patterns and root causes.
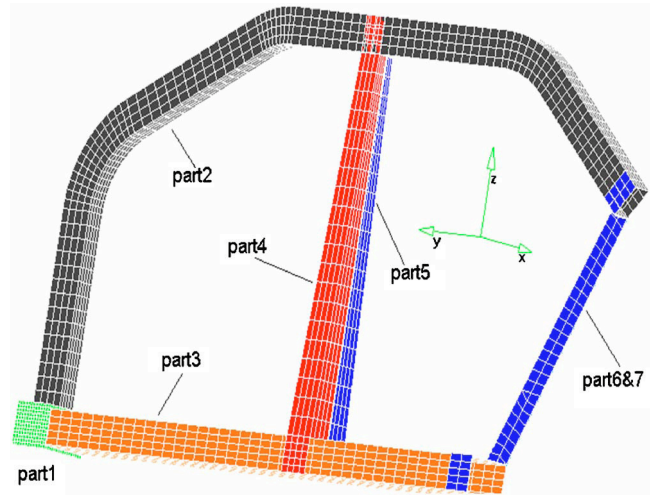

Fig. 2 Finite element model for an automotive side frame structure

4 Case Study

A case study is presented in this section to illustrate the applicability of the proposed methodology in a real assembly product. The product in this case study is a side frame structure of a car. Even though the compliant assembly usually is composed of sheet metal parts and this structure consists of some sheet metal parts and some nonsheet metal parts, the proposed method can still be applied to conduct sensitivity analysis considering the deformation of the nonsheet parts only due to assembly mechanical forces. Pattern, component and station sensitivities are illustrated through this example.

Figure 2 shows a simplified finite element model for the side frame structure. There are seven parts in this structure. Part 1 is a 3 mm thick hollow block. Part 2 is a hydroformed rail and its thickness is 1 mm. Part 3 is fabricated by extrusion, and its thickness is 2 mm. Parts 4–7 are stamped sheet parts, and their thicknesses are 1 mm. All the parts in this assembly are made of steel.

As shown in Fig. 2, there are seven parts on this structure. These parts are welded together at three stations. Parts 2 and 3 are welded to part 1 at the first station. Parts 4 and 5 are added at the second station. Parts 6 and 7 are added to form the final assembly at the third station. After the assembly operations at the third station, the deviations of nine points along the whole frame are measured in Y (out of plane) and Z (up-down) directions.

At each station, the method of influence coefficient is applied to obtain the sensitivity matrix, assuming that all the welding points have some variation in both the Y and Z directions. Based on calculations of the sensitivity matrices at the station level shown as Eq. (1), a state-space model in recursive form shown as Eq. (7) is established for this three-station assembly system and the corresponding matrices are obtained.

Based on the definition of the component sensitivity index (Eq. (15)), Table 2 shows the results of the sensitivity analysis for the

Table 2 Summary of component sensitivity analysis

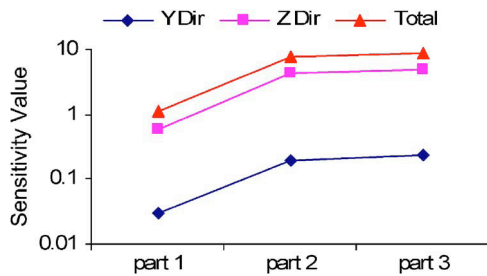| Part No.	Y direction	Z direction	Total
1	0.03	0.2	0.23
2	0.6	4.4	5.0
3	1.1	7.9	9.0
4	0.4×10^{-5}	1.0×10^{-5}	1.4×10^{-5}
5	0.3×10^{-5}	1.7×10^{-5}	2.0×10^{-5}
6	0.1×10^{-5}	0.9×10^{-5}	1.0×10^{-5}
7	0.1×10^{-5}	1.6×10^{-5}	1.7×10^{-5}

Fig. 3 Component sensitivity for Parts 1–3

seven parts/components in the assembly. The sensitivity in the Y direction is the sensitivity of the measurement point's variance in the Y and Z directions to the variance of the source points on a part in Y direction. Similarly, the sensitivity in Z direction is the sensitivity of the measurement point's variance in all the Y and Z directions to the variance of the source points on a part in Z direction. The total sensitivity analysis includes the variance of both Y and Z direction on the source points of a part.

From the summary of component sensitivity analysis for each part in Table 2, it can be seen that the sensitivities for Parts 4–7 are much smaller than those for the other three parts. One reason is that these parts are less stiff than the corresponding subassembly at their respective assembly station, which is mainly due to the geometrical structures of the parts and the assembly process requirements, such as fixture position and welding position. For instance, parts 4–7 are more compliant (flexible) than the subassembly consisting of part 1–3. Another reason is that the parts with small sensitivities are only involved into welding once during the assembly process. Therefore, their variation has fewer opportunities to propagate to the final assembly. For example, parts 4 and 5 are welded to become a subassembly only at the second station. In contrast, parts 2 and 3 are involved into the assembly process at all the three stations.

Since parts 1–3 have more significant effects than the other parts on the dimensional variance of the final assembly, the component sensitivity analysis for these three parts is plotted in Fig. 3. In Fig. 3, the sensitivities for these three parts are grouped into the sensitivity in the Y direction, Z direction, and the sum of both directions. The y-axis in the plot is the logarithmic sensitivities for these parts.

As shown in Fig. 3, the sensitivities in the Z direction are greater than those in the Y direction for all the parts. The reason is that the stiffness of these parts in the Z direction is greater than that in the Y direction during the assembly process. For example, from the geometric structure of part 3, the stiffness in Y direction should be the same as that in the Z direction. However, the difference in stiffness is because part 3 is fixed by three fixtures in the Z direction and two in the Y direction.

Because of the big sensitivities of parts 2 and 3, a sensitivity analysis for the patterns of these two parts was conducted. It is assumed that three patterns, bending about the y-axis, bending about the z axis and twisting, are the major concerns for the dimensional quality of the parts. Table 3 summarizes the analysis results for the three patterns of each part.

Table 4 Eigenvalues related to parts 2 and 3

Part No.	Eigenvalues	
	Largest	Smallest
2	5.3	0.2
3	11.3	0.27

As shown in Table 3, twisting and bending variation patterns about the y axis have a greater sensitivity than the bending about the z-axis. This effect is reasonable because the bending pattern about z-axis induces the variation in y-axis in which direction parts 2 and 3 have small sensitivities.

Based on the proposed method, the ranges of the sensitivities for parts 2 and 3 and their patterns can be obtained. The largest and smallest eigenvalues of the corresponding $\gamma^T \gamma$ matrices for parts 2 and 3 are listed in Table 4.

For patterns on part 2, the sensitivities are 1.0, 1.9, and 0.3, which are within the range of (0.2, 5.3). Additionally, the component sensitivity for part 2 is 5.0, which also is within the range of the (0.2, 5.3) as expected. Similarly, the pattern and component sensitivities of part 3 are also within the range of (0.27, 11.3). It is proved again that the proposed method for the range of the sensitivity is valid. It can be noted that the range for part 3 is bigger than that for part 2. As shown before, these ranges related to $index_{prn}$, defined in a previous section, depend on many factors, such as material, geometries, fixtures, and assembly sequences of components.

The sensitivity analysis results for stations are summarized in Table 5. As shown in Table 5, the component variation at the second station has the most significant impacts on the dimensional variation of the final assembly. In contrast, the component variation at the first station has the least effects on the dimensional variation of the final assembly.

In summary, product-oriented sensitivity is related to the geometry and material properties that determine the stiffness of the components and assembly process information, such as fixturing schemes, welding points, and assembly sequence, and it is also related to the measurement points on the final assembly.

5 Conclusions

In the paper, a set of product-oriented sensitivity indices has been defined for multistation compliant assemblies from different aspects. These indices are important to evaluate the robustness of parts, to allocate tolerances to components, and to improve the dimensional quality of a product. The component sensitivity analysis can effectively evaluate the potential contribution of a component to the product dimensional quality. The pattern sensitivity analysis for variation patterns of a part plays an important role in the process diagnosis and root cause identification due to the inherent relationship between patterns and root causes. The component sensitivity and the station sensitivity analyses evaluate the importance of a component and a station on the product variation propagation, respectively. In addition, a method has also been proposed to obtain the ranges, maximum, and minimum for all the sensitivity indices. The importance of these ranges is in that they can be used to estimate the sensitivities without any information about the incoming variation. In other words, the estimation of the

Table 3 Sensitivity analysis for the patterns of parts 2 and 3

Part No.	Pattern sensitivity		
	Twisting	Bending (y)	Bending (z)
2	1.02	1.9	0.3
3	1.7	1.6	0.5

Table 5 Station sensitivity analysis

Station	Sensitivity
1	2.35
2	26.67
3	12.72

sensitivities is independent of the input variation. This independence is necessary and helpful in most cases at the design stage when limited information for the components variation is available. Finally, a case study has been conducted to evaluate the definitions of these sensitivities.

Acknowledgment

Nomenclature

V_w = dimensional deviation vector for the key product characteristics (KPC) of an assembly

V_u = incoming dimensional deviation vector for the components of an assembly

X_i = state vector ($\mathbf{n}$ vector), which represents the dimensional deviation of the source points at the ith station in a global coordinate system

Y_i = output vector ($\mathbf{m}$ vector) for the key measurement points of the assembly at the ith station

A_i = state matrix ($n \times n$ matrix)

B_i = input matrix ($n \times r$ matrix)

C_i = observation matrix ($m \times n$ matrix)

S_i = sensitivity matrix at the ith station ($n \times n$ matrix)

D_i = deformation matrix before assembly at the ith station ($n \times n$ matrix)

M_i = relocating matrix at the ith station ($n \times n$ matrix)

U_i^{3-2-1} = deviation vector of 3-2-1 fixtures at the ith station

U_i^{n-3} = deviation vector for the n-2-1 ($n > 3$) fixtures at the ith station

U_i^g = deviation vector for the welding guns at the ith station

W_i = disturbance vector at the ith station

E_i = noise vector at the ith station

Σ_y = covariance matrix of measurement points on final assemblies

Σ_{U_i} = covariance matrix of the fixtures at the ith station

Σ_{X_0} = covariance matrix for the source points on all the incoming parts

$\Sigma_{X_{p\_i}}$ = Covariance matrix for the source points on the ith part of an assembly

$\Sigma_{X_{c\_ij}}$ = covariance matrix for the source points on the jth component at the ith station

λ_{ij} = jth eigenvalue of the covariance matrix $\Sigma_{X_{p\_i}}$

P_{ij} = normalized jth eigenvector of the covariance matrix $\Sigma_{X_{p\_i}}$

$\sigma_{Y_i}^2$ = variance of the ith measurement point on the final assembly

m_i = number of source points for the ith part

d = number of measurement points for the final assembly

k = number of parts in the system

N = number of stations in the system

$\mathsf{s}_r, \mathsf{s}_1$ = minimum, maximum eigenvalues of the matrix $\gamma(1)_i^T \gamma(1)_i$

η_r, η_1 = minimum, maximum singular values of the matrix $\gamma(1)_i$

S_{prn_ij} = product-oriented sensitivity index for the jth variation pattern of the ith part

S_{prt_i} = product-oriented component sensitivity index for the ith part

S_{stn_i} = product-oriented station sensitivity for the ith station of a system

References

[1] Taguchi, G., and Wu, Y., 1979, "*Introduction to Off-Line Quality Control*, Central Japan Quality Control Association, Meieki Nakamura-Ku Magaya, Japan.

[2] Arora, J. S., and Haug, E. J., 1979, "Methods of Design Sensitivity Analysis in Structural Optimization," AIAA J., **17**(9), pp. 970–974.

[3] Beltracchi, T. J., and Gabriele, G. A., 1988, "A RQP Based Method for Estimating Parameter Sensitivity Derivatives," *Proceedings of Design Automation Conference*, ASME, New York, Vol. 14, pp. 155–164.

[4] Phadke, M. S., 1989, *Quality Engineering Using Robust Design*, Prentice Hall, Englewood Cliffs, NJ.

[5] Parkinson, A., Sorensen, C., Free, J., and Canfield, B., 1990, "Tolerances and Robustness in Engineering Design Optimization," *Proceedings of 1990 ASME Design Automation Conference*, Sept., Chicago, ASME, New York, Vol. 2, pp. 121–128.

[6] Parkinson, A., Sorensen, C., and Pourhassan, N., 1993, "A General Approach for Robust Optimal Design," ASME J. Mech. Des., **115**, pp. 74–80.

[7] Parkinson, A., 1995, "Robust Mechanical Design Using Engineering Models," ASME J. Mech. Des., **117**, pp. 48–54.

[8] Chen, W., Allen, J. K., Tsui, K.-L., and Mistree, F., 1996, "A Procedure for Robust Design: Minimizing Variations Caused by Noise Factors and Control Factors," ASME J. Mech. Des., **118**, pp. 478–485.

[9] Ting, K., and Long, Y., 1996, "Performance Quality and Tolerance Sensitivity of Mechanisms," ASME J. Mech. Des., **118**(1), pp. 144–150.

[10] Cai, W., Hu, S. J., and Yuan, J. X., 1997, "A Variational Method of Robust Fixture Configuration Design for 3-D Workpieces," ASME J. Manuf. Sci. Eng., **119**, pp. 593–602.

[11] Gao, J., Chase, K. W., and Magleby, S. P., "Global Coordinate Method for Determining Sensitivity in Assembly Tolerance Analysis," http://adcats.et.byu.edu/WWW/Publication/98-3/Paper6_2col_6=29=98.html

[12] Söderberg, R., and Lindkvist, L., 1999, "Computer Aided Assembly Robustness Evaluation," J. Eng. Design, **10**(2), pp. 165–181.

[13] Thornton, A. C., 2000, "Quantitative Selection of Variation Reduction Plans," ASME J. Mech. Des., **122**, pp. 185–193.

[14] Ding, Y., Ceglarek, D., and Shi, J., 2002, "Design Evaluation of Multi-Station Assembly Processes by Using State Space Approach," ASME J. Mech. Des., **124**(3), pp. 408–418.

[15] Hu, S. J., Webbink, R., Lee, J., and Long, Y., 2003, "Robustness Evaluation for Compliant Assembly Systems," ASME J. Mech. Des., **125**(2), pp. 262–267.

[16] Caro, S., Bennis, F., and Wenger, P., 2005, "Tolerance Synthesis of Mechanisms: A Robust Design Approach," ASME J. Mech. Des., **127**(1), pp. 86–94.

[17] Lee, B., and Saitou, K., 2006, "Three-Dimensional Assembly Synthesis for Robust Dimensional Integrity Based on Screw Theory," ASME J. Mech. Des., **128**(1), pp. 243–251.

[18] Lindvist, L., and Söderberg, R., 2003, "Computer Aided Tolerance Chain and Sensitvity Analysis," J. Eng. Design, **14**(1), pp. 17–39.

[19] Ceglarek, D., and Shi, J., 1998, "Variation Design Evaluation of Sheet Metal Joints for Dimensional Integrity," ASME J. Manuf. Sci. Eng., **120**, pp. 452–460.

[20] Wang, M. Y., 1999, "An Optimum Design Approach to Fixture Synthesis for 3D Workpieces," Trans. North Am. Manuf. Res. Inst. SME, **XXVII**, pp. 209–214.

[21] Liu, S. C., Hu, S. J., and Woo, T. C., 1996, "Tolerance Analysis for Sheet Metal Assemblies," ASME J. Mech. Des., **118**, pp. 62–67.

[22] Liu, S. C., and Hu, S. J., 1997, "Variation Simulation for Deformable Sheet Metal Assemblies Using Finite Element Methods," ASME J. Manuf. Sci. Eng., **119**, pp. 368–374.

[23] Camelio, J., Hu, S. J., and Ceglarek, D., 2003, "Modeling Variation Propagation of Multi-Station Assembly Systems With Compliant Parts," ASME J. Mech. Des., **125**(4), pp. 673–681.

[24] Hu, S. J., and Wu, S. M., 1992, "Identifying Root Causes of Variation in Automobile Body Assembly Using Principal Component Analysis," Trans. North Am. Manuf. Res. Inst. SME, **20**, pp. 311–316.

[25] Ceglarek, D., and Shi, J., 1996, "Fixture Failure Diagnosis for Autobody Assembly Using Pattern Recognition," ASME J. Eng. Ind., **118**, pp. 55–66.

[26] Camelio, J., and Hu, S. J., 2004, "Multiple Fault Diagnosis for Sheet Metal Fixtures Using Designated Component Analysis," ASME J. Manuf. Sci. Eng., **126**(1), pp. 91–97.

[27] Camelio, J., Hu, S. J., and Marin, P. S., 2004, "Compliant Assembly Variation Analysis Using Geometric Covariance," ASME J. Manuf. Sci. Eng., **126**(2), pp. 355–360.