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Modeling Variation Propagation of Multi-Station Assembly Systems With Compliant Parts

Products made of compliant sheet metals are widely used in automotive, aerospace, appliance and electronics industries. One of the most important challenges for the assembly process with compliant parts is dimensional quality, which affects product functionality and performance. This paper develops a methodology to evaluate the dimensional variation propagation in a multi-station compliant assembly system based on linear mechanics and a state space representation. Three sources of variation: part variation, fixture variation and welding gun variation are analyzed. The proposed method is illustrated through a case study on an automotive body assembly process. [DOI: 10.1115/1.1631574]

1 Introduction

Compliant sheet metal assembly is a manufacturing process where two or more sheet metal parts are joined together using various joining techniques. The result of this process is a subassembly or a final product. One of the most important challenges for sheet metal assembly is the understanding of how dimensional variation propagates [1,2]. Due to the variability of the parts, fixtures and joining methods in each station and their interactions, a sheet metal assembly process can be considered as a variation "stack up" process.

Dimensional variation can stem from both the design and manufacture of a product. Since some manufacturing induced variation is inevitable, it is important to minimize the level of inherent dimensional variation caused by product and process design. Many of the problems associated with dimensional accuracy occur because the capability of the manufacturing process is not considered when designing the product and process. These problems may affect the final product functionality and process performance. For example, large product dimensional variation in an automotive body assembly process may cause product problems such as water leakage and wind noise, as well as process difficulties such as fitting problems in subsequent operations. Clearly, reducing dimensional variation in an assembly process is of critical importance to improve the final product quality. In addition, a better understanding of a process behavior can also bring a reduction in the time needed to launch a new manufacturing system. Early and accurate evaluations of inherent process variation are crucial factors in determining the final dimensional variation of an assembled product.

In recent years, the importance of dimensional variation has been observed by an increasing amount of research conducted in the area of sheet metal assembly processes. Since Takezawa [3] observation that for compliant sheet metal assemblies the traditional additive theorem of variance is no longer valid, several models have been proposed to represent the variation propagation on assembly processes. The models developed can be grouped into four different categories, depending on whether the model is for a single station or a multi-station process, or if the model considers rigid or compliant parts. Station level models treat the assembly process as if it is conducted in one step. In contrast, multi-station models analyze the process recursively as the assembly is moved from one station to the next. Rigid part models do not consider part deformation during assembly so that the part and tooling variation can be solely represented by kinematic relationships. Compliant part models consider the possible deformation of the parts during the assembly process. The models include a force analysis that take into consideration the stiffness of each part and the forces applied by each tool.

Dimensional variation modeling and analysis for multi-station manufacturing processes has been developed mainly for rigid parts. Multi-station assembly processes with rigid parts cover a large number of currently used processes such as power-train assembly and general assembly in automotive industry. However, a large group of multi-station assembly processes consider non-rigid parts. For example, 37% of all assembly stations in automotive body structure manufacturing assemble nonrigid parts [4]. Variation propagation analysis for a multi station assembly process introduces new modeling challenges. In comparison to the station level approach, it is necessary to define an appropriate variation representation in order to track the variation propagation from station to station. The variation simulation process is sequential, i.e., to estimate the variation at *station i*, it is necessary to know the variation at *station i−1*. Moreover, there is a station-to-station interaction introduced by the release of holding fixtures and the use of new fixtures in subsequent stations. Finally, compliant assembly variation analysis requires applying finite element methods to calculate the deformation after assembly. Therefore, the number of calculations increases with the number of stations.

Recent publications in each of these areas are summarized in Table 1. As can be seen, most of the dimensional variation analysis has been conducted for single station or multi-station rigid part assembly and some work exists at station level for compliant assembly.

At station level, Liu et al. [5] and Liu and Hu [6] proposed a model to analyze the effect of deformation and springback on assembly variation by applying linear mechanics and statistics. Using finite element methods (FEM), they constructed a sensitivity matrix for compliant parts of complex shapes. The sensitivity matrix establishes the linear relationship between the incoming part deviation and the output assembly deviation. Long and Hu [7] extended this model to a unified model for variation simulation by considering part variation and locating fixture variation. Shiu et al. [4] presented a simplified flexible beam representation of body structures. Huang and Ceglarek [8] presented a discrete-cosine-transformation (DCT) based decomposition method for modeling and control of compliant assemblies form error. The method decomposes the dimensional error field into a series of independent error modes.

At multi-station level, Lawless et al. [9] proposed a method called Variation Driver Analysis using time series analysis. The method is based in tracking the characteristics of individual parts as they pass through multiple stations using autoregressive mod-

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Table 1 Recent publications in dimensional variation

	Rigid Parts	Compliant Parts
Single Station Level	Lee and Woo [21] Chase and Parkinson [22] Etc.	Liu et al. [5] Cai et al. [20] Liu and Hu [6] Shiu et al. [4] Long and Hu [7] Huang and Ceglarek [8]
Multi-Station Level	Shiu et al. [23] Mantripragada and Whitney [10] Jin and Shi [11] Ding et al. [12]	To be developed in this Paper

els. Mantripragada and Whitney [10] proposed a variation propagation model using state transition models. The proposed model considers rigid parts and the state space vector can be fully described by a translation and re-orientation. The state transition model allows the application of control systems theory to the analysis of multi-station assembly system. Jin and Shi [11] proposed a state space modeling approach for dimensional control for in plane motion of rigid parts in a sheet metal assembly process, where the state equation considers two types of dimensional variation, the part error itself and the fixture error. Ding et al. [12,13] developed a complete state space model for variation in the plane of rigidity for rigid components.

Comparatively, little research has been done in multi-station systems considering compliant, non-rigid parts. Liu and Hu [14] developed a model to evaluate the spot weld sequence in sheet metal assembly. This model considered a process where welding was carried out in multiple stages. Chang and Gossard [15] presented a graphic approach for multi-station assembly of compliant parts. However, there are no analytical models for variation analysis in multi-station compliant assembly systems.

It is critical to develop realistic models for sheet metal assembly process that consider compliant parts and also include the station-to-station interaction in multi-station assembly systems. Such models can be quite useful during both the design and launch stage of the manufacturing system. During design, such models can be used to predict product variation so that changes in parts or processes can be made early. During the launch of the manufacturing system, such models can aid in the diagnosis of root causes of variation [16,17]. The purpose of this paper is to present a methodology for modeling the impact of part and tooling variation on the dimensional quality on a multi-station assembly system with compliant sheet metal parts and study how variation propagates from different subassemblies to the final product.

The remainder of this paper is organized as follows. Section 2 presents the model for multi-station variation analysis using a state space representation. The model is developed for three different sources of variation: part variation, fixture variation and welding gun variation. In Sec. 3, a simplified example for automotive sheet metal assembly variation analysis is presented. Finally, Sec. 4 draws conclusions.

2 Multi-Station Assembly Model

The automotive body assembly process will be used to develop the methodology for modeling and analyzing dimensional variation propagation in multi-station systems. However, the developed model can be generalized into other multi-station assembly process with compliant parts such as appliances or furniture manufacturing.

An automotive body assembly process is a multi-leveled hierarchical process, in which sheet metal parts are joined together to form a subassembly [2,18]. During the assembly process, each part or subassembly becomes an input for the subsequent stations. While parts move from one station to another, dimensional variation of the parts and subassemblies propagates through the system. The station-to-station interactions cause an increase or sometimes decrease of the dimensional variation.

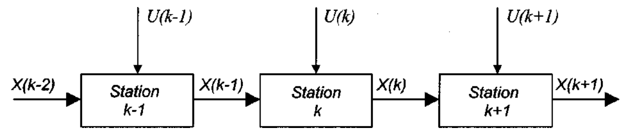

Fig. 1 Deviation propagation in a multi-station system

An assembly process can be considered a discrete-time dynamical system, where the independent variable time can represent the station location. Then, a state space representation can be developed to illustrate the part deviation [10–12]. In station k, part deviation after assembly operations is function of the input parts deviation and tooling deviation, as shown in Fig. 1. Then, part deviation can be calculated by the state equation, Eq. (1), if function f is known. In addition, extra measurement point deviation can be calculated using the observation equation, Eq. (2), for a given function g. The objective of the model is to define the appropriate functions f and g.

$$\mathbf{X}(k)=f_k(\mathbf{X}(k-1),\mathbf{U}(k)) \tag{1}$$

$$\mathbf{Y}(k)=g_k(\mathbf{X}(k)) \tag{2}$$

where $\mathbf{X}(k)$ represents the part deviation for every part in the assembly at station k, $\mathbf{U}(k)$ the tooling deviation at station k and $\mathbf{Y}(k)$ the key characteristic points deviation at station k.

The developed methodology will be presented in two steps. First, a short description of a single station model will be reviewed and adapted to our methodology. Second, the multi-station model will be developed based on the single station model and station-to-station variation propagation model.

2.1 Single Station Assembly Modeling. Traditionally, modeling of dimensional variation propagation for a single station assumes that all the process operations occur simultaneously, i.e., sequence of operations and interactions between operations are not taken into considerations. In this paper, the variation modeling approach at the station level is based on the mechanistic simulation method developed by Liu and Hu [6]. This procedure assumes that: sheet deformation is in the linear elastic range; the material is isotropic; fixture and welding gun are rigid; there is no thermal deformation and stiffness matrix remains constant for non-nominal part shapes. Representation of the assembly process with compliant parts is illustrated in Fig. 2 and can be described in the following steps:

1. Part loading and locating operation (Fig. 2*a*)
2. Part holding operation (Fig. 2*b*)
3. Part joining operation, such as spot welding (Fig. 2*c*)

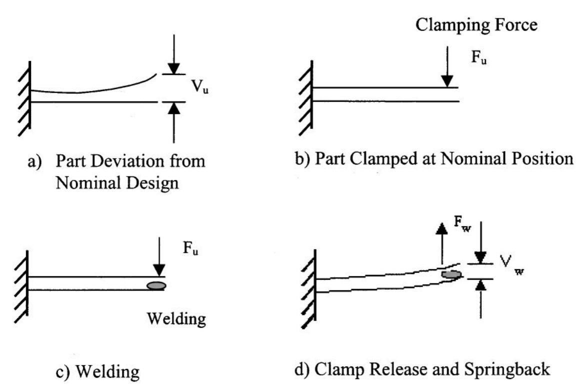

Fig. 2 Sheet metal assembly process

4. Part unloading, the clamp is released and the subassembly springback (Fig. 2*d*).

The modeling of these four steps is presented as follows:

Step 1 The parts are loaded and located in the station using a locating scheme (Fig. 2*a*). After locating part 1, it has a deviation $\mathbf{V_u}$ from the nominal part shape. If more than one source of deviation is present, a vector $\{\mathbf{V_u}\}$ will represent the deviation. Index u refers to unwelded parts.

Step 2 Part 1 deviation ($\mathbf{V_u}$) is closed by a welding gun or a fixture applying a force $\mathbf{F_u}$ (Fig. 2*b*). If there is more than one source of variation, $\{\mathbf{F_u}\}$ will be a vector. Then considering a part stiffness matrix $\mathbf{K_u}$ the force required to close the gap $\mathbf{V_u}$ will be given by Eq. (3).

$$\{\mathbf{F}_u\}=[\mathbf{K}_u]\cdot\{\mathbf{V}_u\} \tag{3}$$

Step 3 The parts are joined together while the force $\mathbf{F_u}$ is still being applied (Fig. 2*c*).

Step 4 The welding gun/fixture is removed (Fig. 2*d*). After removing the forces applied by the clamping system, the new assembled structure will springback. The springback position is determined assuming that a force ($\mathbf{F_w}$) of the same magnitude of the clamping forces ($\mathbf{F_u}$) but in opposite direction is applied over the nominal welded structure. Knowing the assembly stiffness matrix ($\mathbf{K_w}$), the value of the springback variation ($\mathbf{V_w}$) can be determined using Eq. (4)–(7),

$$\{\mathbf{F}_w\}=[\mathbf{K}_w]\cdot\{\mathbf{V}_w\} \tag{4}$$

$$\{\mathbf{F}_u\}=\{\mathbf{F}_w\} \tag{5}$$

$$\{\mathbf{V}_w\}=[\mathbf{K}_w]^{-1}\cdot[\mathbf{K}_u]\cdot\{\mathbf{V}_u\} \tag{6}$$

$$\{\mathbf{V}_w\}=[\mathbf{S}_{uw}]\cdot\{\mathbf{V}_u\} \tag{7}$$

where, $[\mathbf{S_{uw}}]$ is the sensitivity matrix, which represents how sensitive is the output assembly deviation to an input part deviation, where index u represents the input source of variation and w the output measurement points. As a result, the springback of the assembly can be represented by the mechanistic variation model as,

$$\mathbf{V}_w=\mathbf{S}\cdot\mathbf{V}_u \tag{8}$$

The sensitivity matrix $\mathbf{S}$ can be determined using the method of influence coefficients as presented in Liu and Hu [6]. Then, considering a linear relationship between the incoming parts deviation and the final assembly deviation for compliant parts at the station level and using finite element analysis, it is possible to construct the sensitivity matrix for a specific station configuration. Finite element methods are used to obtain the stiffness matrices for parts of complex shape.

2.2 Multi-Station Assembly Modeling. Considering the variation propagation process as a linear time varying discrete time system, where the variable time represents station location, a state space representation can be used to model the multi-station assembly process. Therefore, the dimensional deviation of the assembly parts can be represented by the following equations:

$$\mathbf{X}(k)=\mathbf{A}(k)\cdot\mathbf{X}(k-1)+\mathbf{B}(k)\cdot\mathbf{U}(k)+\mathbf{W}(k) \tag{9}$$

$$\mathbf{Y}(k)=\mathbf{C}(k)\cdot\mathbf{X}(k)+\mathbf{W}(k) \tag{10}$$

where, $\mathbf{X}(k)$ is the state vector, $\mathbf{A}(k)$ is the state matrix, $\mathbf{B}(k)$ is the input matrix, $\mathbf{U}(k)$ is the input vector, $\mathbf{C}(k)$ is the observation matrix and $\mathbf{W}(k)$ is noise. The following sections will develop the expressions for $\mathbf{X}(k)$, $\mathbf{A}(k)$, $\mathbf{B}(k)$ and $\mathbf{U}(k)$.

2.2.1 State Vector, X(k). The discrete system state vector is a set of variables that allow representation of the system behavior. The state vector will include the representation of all the parts/subassemblies at each station in the system. Compliant sheet metal parts require more than two points to represent a state compared with the just two points required for rigid body representation. In fact, to completely describe the real part shape it will be necessary to know an infinite number of points, similar to a mesh in FE analysis.

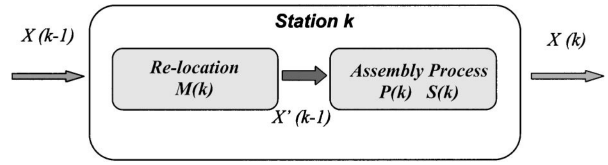

Fig. 3 Re-location and assembly process

There is a balance between the accuracy of the model, the time necessary to conduct the simulation and the size of the matrices. The number of points selected to represent a part will depend of the complexity of the parts and the accuracy necessary. Therefore, a limited number of points are used to analyze how variation propagates through the assembly line. The relevant points required to represent a compliant part state are: the part deviation on the welding positions or welding locating points (WLP), the fixture points or principal locating points (PLP) and any additional measurement point or measurement locating points (MLP).

Due to the use of finite element methods (FEM), a mesh is generated defining these particular points as nodes among the complete mesh. Then, for compliant part j, at station k, we will have a state vector represented as:

$$\mathbf{X}^j(k)=[X_{WLP_1}\ \cdots\ X_{WLP_i}\ X_{PLP_1}\ \cdots\ X_{PLP_j}\ X_{MLP_1}\ \cdots\ X_{MLP_l}]^T \tag{11}$$

Thus, for an assembly of n parts, the state vector at station k, will be,

$$X(k)=\begin{bmatrix} X^1(k) \\ X^2(k) \\ \vdots \\ X^n(k) \end{bmatrix} \tag{12}$$

2.2.2 State Transition Matrix, A(k), With No Tooling Variation. Using a modified mechanistic variation simulation method, it is possible to define the relation among the input parts deviation and the output subassembly deviation. This relation is the result of three consecutive operations (Fig. 3). First, incoming parts are re-located in the station using a 3-2-1 fixture layout. Second, the part is deformed when the welding guns and additional clamps on the primary plane are closed and the parts are welded to produce a subassembly. Finally, the welding guns and fixture are released causing sprinback.

Each of these operations can be represented in a matrix form. The re-location/re-orientation effect is defined by the matrix $\mathbf{M}$. The part deformation before welding is represented by matrix $\mathbf{P}$. The springback for unit deviations can be obtained from matrix $\mathbf{S}$. The following section will define matrices $\mathbf{M}$, $\mathbf{P}$, $\mathbf{S}$ and their relation with the state transition matrix $\mathbf{A}$ (Eq. (13))

$$\mathbf{A}(k)=f(\mathbf{M}(K),\mathbf{P}(k),\mathbf{S}(k)) \tag{13}$$

Re-location matrix: As presented previously, the state space vector for station k represents the components/subassemblies deviation after station k in a global coordinate system. At each station the components/subassemblies are located considering a 3-2-1 locating scheme. Considering the example of a 2D-beam in Fig. 4, the 3-2-1 locating scheme will be equivalent to add 2 locators in z-direction and 1 locator in the x-direction. Then, we can study the relocation effect in the xz plane. In Fig. 4, part j is located with locators P_1 and P_2 at station $k-1$. After the assembly process at station $k-1$ is finished, the fixture P_1 and P_2 are released and the part is moved to the next station (station k). At station k a new locating scheme is utilized, the components/

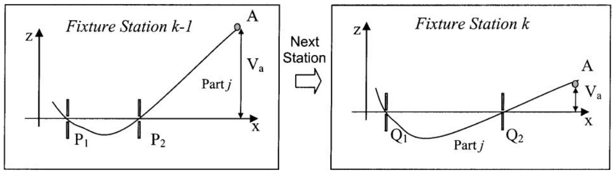

Fig. 4 Locating process from station $(k-1)$ to station (k)

subassemblies will be re-located using a new set of locators, Q_1 and Q_2. The state vector will be modified accordingly.

Considering the part deviation in the z-direction at station $k-1$ in points Q_1 and Q_2, the position of point A (part j) at station k will change due to the re-location process. The coordinates of point A can be obtained using homogeneous transformation. Assuming small deviations, the displacements in the x-direction can be neglected. Therefore, the linear relationship for the part deviation in the z-direction due to re-orientation will be,

$$\Delta X_A^j = \underbrace{\left[\frac{x_A}{x_{Q_2}-x_{Q_1}} - 1 \quad \frac{-x_A}{x_{Q_2}-x_{Q_1}} \right]}_{\mathbf{M}_A^j(k)} \cdot \begin{bmatrix} \Delta z_{Q_1} \\ \Delta z_{Q_2} \end{bmatrix} \tag{14}$$

where $\mathbf{M}_A^j(k)$ is the re-location matrix for point A in part j at station (k).

Extending the same concept for each point of the state vector and a *3-2-1* locating scheme in 3D, the state space vector after relocating in station k will be,

$$\mathbf{X}'(k-1) = \mathbf{X}(k-1) + \mathbf{M}(k)\cdot\mathbf{X}(k-1) \tag{15}$$

where, the relocating matrix M has the form,

$$M(k)=\begin{bmatrix} \overbrace{\begin{matrix} 0 & M_1^1 & 0 \\ \vdots & M_2^1 & \vdots \\ \vdots & \vdots & \vdots \\ 0 & M_{m_1}^1 & 0 \end{matrix}}^{Part_1} & 0 & 0 & 0 \\ 0 & \overbrace{\begin{matrix} 0 & M_1^2 & 0 \\ \vdots & M_2^2 & \vdots \\ \vdots & \vdots & \vdots \\ 0 & M_{m_2}^2 & 0 \end{matrix}}^{Part_2} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \overbrace{\begin{matrix} 0 & M_1^n & 0 \\ \vdots & M_2^n & \vdots \\ \vdots & \vdots & \vdots \\ 0 & M_{m_n}^n & 0 \end{matrix}}^{Part_n} \end{bmatrix}_{(nxn)} \tag{16}$$

where, n is the number of elements in state vector and m_i is the number of key points in part/subassembly i. The zeros in the matrix $\mathbf{M}(k)$ correspond to the null impact of others subassembly locators over one determined part or subassembly.

The effect of the additional clamps in the primary plane (*N-2-1* locating scheme) is not considered in the locating process. Once the *3-2-1* locating scheme is used to completely locate the part in a station, any additional clamps (*N-3*) will deform the parts. Therefore, the analysis of the additional clamps will be included in the matrices **P** and **S**.

Deformation and sensitivity matrix: Using method of influence coefficients [6], it is possible to obtain the sensitivity matrix for a specific station k. It should be noticed that sources of variation at station k have a null effect over parts that are not being assembled at that station. Therefore, the elements of matrix S for the parts that do not participate in the assembly in that station are equal to zero. Then, the matrix S has the form,

$$\mathbf{S}(k)=\begin{bmatrix} \mathbf{S}_{Subassembly_1} & 0 & 0 & 0 \\ 0 & \mathbf{S}_{Subassembly_2} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \mathbf{S}_{Subassembly_n} \end{bmatrix} \tag{17}$$

The method of influence coefficients [6] assumed a specific shape of the subassembly components for a given source of variation vector. Deformation matrix **P** represents this assumed components deformation. Considering a different incoming part shape only the net effect of the assembly process must be added to the incoming part variation. To calculate the net effect, Matrix **P** must be subtracted from the sensitivity matrix **S**. Therefore, the relation between the output assembly deviation and the incoming part deviation can be expressed as:

$$\mathbf{V}_w=(S-P)\cdot\mathbf{V}_u+\mathbf{V}_u$$

Using method of influence coefficients; it is possible to obtain the deformation matrix **P** for the assembly components as,

$$\mathbf{P}(k)=\begin{bmatrix}\mathbf{P}_{Part_1} & 0 & 0 & 0\\ 0 & \mathbf{P}_{Part_2} & 0 & 0\\ 0 & 0 & \ddots & 0\\ 0 & 0 & 0 & \mathbf{P}_{Part_n}\end{bmatrix} \tag{18}$$

Finally, using the state space representation and the matrices **M**, **P** and **S**, the model can be expressed by,

$$\mathbf{X}'(k-1)=\mathbf{X}(k-1)+\mathbf{M}(k)\cdot\mathbf{X}(k-1) \tag{19}$$

$$\mathbf{X}(k)=(\mathbf{S}(k)-\mathbf{P}(k))\cdot\mathbf{X}'(k-1)+\mathbf{X}'(k-1)+\mathbf{W}(k) \tag{20}$$

and the state transition matrix will be,

$$\mathbf{A}(k)=(\mathbf{S}(k)-\mathbf{P}(k)+\mathbf{I})\cdot(\mathbf{I}+\mathbf{M}(k)) \tag{21}$$

2.2.3 State Transition Matrix, A(k), With Tooling Variation Part deviation is only one of the variation contributors in sheet metal assembly process. Ceglarek and Shi [1] established that a high percent of all root causes failures for autobody assembly process are due to fixture related problems. Consequently, it is necessary to consider the effect of tooling deviation over the assembly variation. Tooling variation impact can be decomposed into two independent sources of variation: welding gun variation and fixture variation, including locators and clamps.

Welding gun variation: Variation of welding guns has been shown to have a large impact on the final assembly variation [14]. The influence on assembly variation will depend on the welding gun type. In general, three types of welding guns are used in sheet metal assembly, position controlled welding gun, equalized welding gun and force controlled welding gun. In this paper, a position controlled welding gun variation model is presented. Without loss of generality the methodology can be applied to the other two welding gun types.

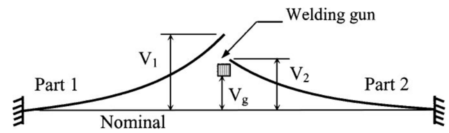

Fig. 5 Position welding gun variation impact

A position controlled welding gun is used to weld two parts at the gun/electrode position. Position controlled welding gun model assumes that the welding gun can apply a sufficient force over the part to close the gap between the part deviation and its electrode position. As shown in Fig. 5, part 1 has a deviation of v_1 and part 2 has a deviation of v_2. In addition, the welding gun has a deviation from the nominal v_g. The force required to close the gap in part 1 and 2 will be:

$$F_1=K_1(v_1-v_g) \tag{22}$$

$$F_2=K_2(v_2-v_g) \tag{23}$$

where K_1 and K_2 is the stiffness of part 1 and part 2 respectively.

The resulting force that will produce springback over the assembly of welded parts 1 and 2 with stiffness K_a will be $F=F_1+F_2$. Therefore, the springback will be,

$$v_a=\frac{F}{K_a}=\frac{K_1}{K_a}\cdot v_1+\frac{K_2}{K_a}\cdot v_2-\frac{(K_1+K_2)}{K_a}\cdot v_g \tag{24}$$

Similarly, using the sensitivity matrix definition,

$$v_a=\mathbf{S}\cdot\begin{bmatrix}v_1-v_g\\ v_2-v_g\end{bmatrix}=S\cdot\begin{bmatrix}v_1\\ v_2\end{bmatrix}-S\cdot\begin{bmatrix}v_g\\ v_g\end{bmatrix}$$

Finally, considering the state space representation, and defining an input vector $\mathbf{U}_g$ for the welding gun deviation, Eq. (20) can be rewritten as:

$$\mathbf{X}(k)=(\mathbf{S}(k)-\mathbf{P}(k)+\mathbf{I})\cdot\mathbf{X}'(k-1)-(\mathbf{S}(k)-\mathbf{P}(k))\cdot\mathbf{U}_g+\mathbf{W}(k) \tag{25}$$

where, $\mathbf{U}_g$, the welding gun deviation vector, has the form,

$$\mathbf{U}_g(k)=\Big[\underbrace{v_{g_1}\ldots v_{g_{p1}}}_{WLP_{part1}}\ 0\ 0\ \underbrace{v_{g_1}\ldots v_{g_{p3}}}_{WLP_{part3}}\ 0\ 0\ \ldots\ \underbrace{v_{g_1}\ldots v_{g_{pn}}}_{WLP_{partn}}\ 0\ 0\Big]^T$$

Fixture variation: The (3-2-1) fixturing principle [19] is a locating principle for a rigid body. However, locating and holding compliant sheet metal workpiece requires a (N-2-1) fixturing principle [20]. The variation model presented considers a decomposition of the fixture variation vector into two sets of fixtures, the (3-2-1) locating fixtures, $\mathbf{U}_{t_{3-2-1}}$ and the $(N-3)$ additional holding fixtures, $\mathbf{U}_{t_{(N-3)}}$. The locating fixture variation effect is considered as a rigid body translation and rotation, and can be obtained by kinematics analysis. The locating fixture variation directly impacts the state space equation at the re-location process. Therefore, Eq. (19) will be:

$$\mathbf{X}'(k-1)=\mathbf{X}(k-1)+\mathbf{M}(k)\cdot\lfloor\mathbf{X}(k-1)-\mathbf{U}_{t_{3-2-1}}(k)\rfloor \tag{26}$$

where $\mathbf{U}_{t_{3-2-1}}$ corresponds to the input vector for the location fixture deviation and has the form,

$$\mathbf{U}_{t_{3-2-1}}(k)=\Big[0\ \underbrace{v_{t_1}\ldots v_{t_{p1}}}_{PLP_{part1}}\ 0\ 0\ \underbrace{v_{t_1}\ldots v_{t_{p3}}}_{PLP_{part3}}\ 0\ \ldots\ 0\ \underbrace{v_{t_1}\ldots v_{t_{pn}}}_{PLP_{partn}}\ 0\Big]^T$$

On the other hand, the additional $(N$-3) clamps can be analyzed equivalently as extra position-controlled welding guns. However, the FEA model does not consider any links at those nodes. Therefore, the clamps apply a force over the part. The force is released after assembly, and then, it produces a springback. The method of influence coefficients proposed by Liu and Hu [6] can

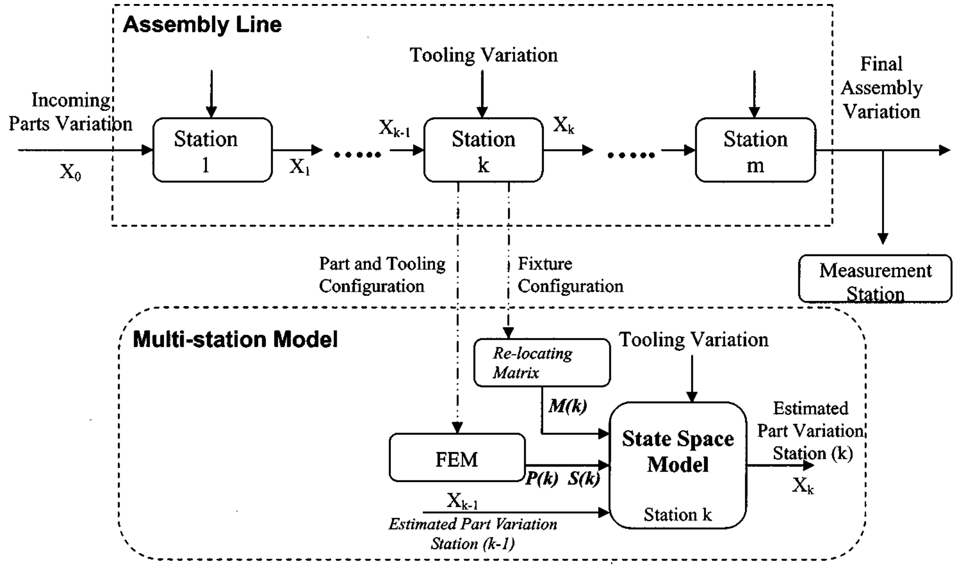

Fig. 6 Multi-station assembly modeling methodology

be applied considering the additional clamps as another source of variation. Finally, Eq. (25) can be rewritten as:

$$\mathbf{X}(k)=(\mathbf{S}(k)-\mathbf{P}(k)+\mathbf{I})\cdot\mathbf{X}'(k-1)-(\mathbf{S}(k)-\mathbf{P}(k))\cdot[\mathbf{U}_g+\mathbf{U}_{t_{(N-3)}}]+\mathbf{W}(k) \qquad (27)$$

2.3 State Space Model for Part and Tooling Variation. Finally, the state space model considering the part variation, fixture variation (N-2-1 fixturing principle) and welding gun variation can be represented by Eq. (28) and (29).

$$\mathbf{X}'(k-1)=\mathbf{X}(k-1)+\mathbf{M}(k)\cdot(\mathbf{X}(k-1)-\mathbf{U}_{t_{3-2-1}}) \qquad (28)$$

$$\mathbf{X}(k)=(\mathbf{S}(k)-\mathbf{P}(k)+\mathbf{I})\cdot\mathbf{X}'(k-1)-(\mathbf{S}(k)-\mathbf{P}(k))\cdot(\mathbf{U}_g+\mathbf{U}_{t_{(N-3)}})+\mathbf{W}(k) \qquad (29)$$

2.4 Methodology. The modeling methodology is applied to study the variation propagation in a multi-station assembly line as shown in Fig. 6. The state space representation for a discrete system requires an independent modeling of each station. First, a state vector must be defined including every component on the final assembly. The state vector must include welding points, locating points and measurement points for each of these components. In addition, for each station, it is necessary to define the locating matrix **M**, using homogeneous transformation; the sensitivity matrix (springback matrix) **S**, using FEM and the method of influence coefficients; and the deformation matrix **P**, using FEM. Finally, knowing the tooling variation and initial part variation and using Eq. (28) and Eq. (29), it is possible to estimate the expected variation for the output subassembly at each station. The process is sequential, in other words, knowing the estimated part variation at station k, it is possible to estimate the part variation at station $k+1$.

The information required to create the state and input matrix may be obtained from the existing assembly line or from the design drawings of a new assembly line. Location matrix **M** will be created using the drawings of each part and the position of the locating fixtures (3-2-1) in each components or subassembly. On the other hand, matrix **P** and **S** will be generated using the finite element model of each assembly station. The finite element model must include the drawings of each component, the location of each fixture and the location of each welding gun. Using this information and the mechanistic simulation method described in section 2.1, matrix **P** and matrix **S** can be defined numerically.

Therefore, the output assembly mean deviation and standard deviation or variance can be estimated applying Monte Carlo simulation. The input variables of the simulation are the initial parts mean deviation and variance and the tooling mean deviation and variance at each station.

3 Application

3.1 Case Study Description. The proposed methodology is illustrated with an example representing the assembly of a car body side, Fig. 7. This process considers three stations and four parts. In station 1, part B and the reinforcement B' are joined together in a parallel assembly. Next, in station 2, the subassembly $(B+B')$ is joined together with part A. Finally, in station 3, part C is joined to the subassembly $(A+(B+B'))$ (Fig. 8).

The simplified model used to represent this assembly process is shown in Fig. 8. Dimensions of the parts, welding points and locating points are included in Fig. 8. The material of every part in the example is mild steel with young modulus $E=20{,}700\ \mathrm{N/mm^2}$ and Poison's ratio $\nu=0.3$. The thickness of the parts is 1 mm. The geometric model has been meshed to create a computational input model. A variety of data has been included in each or the four parts to completely describe the process on each station. The data includes the material properties, boundary con-

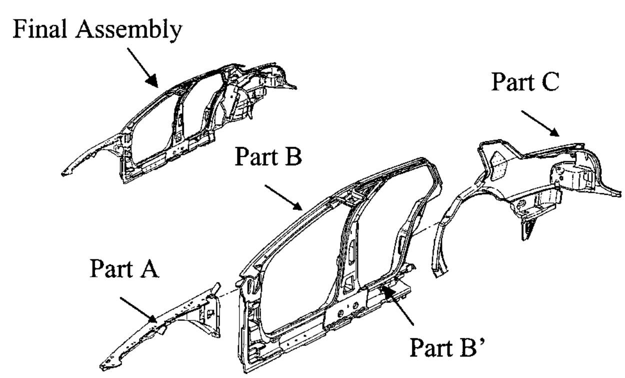

Fig. 7 Right side body structure

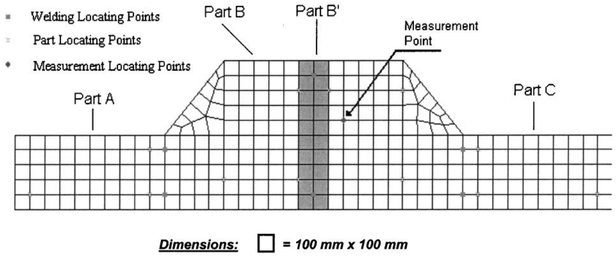

Fig. 8 Mesh of a simplified model for the right side frame

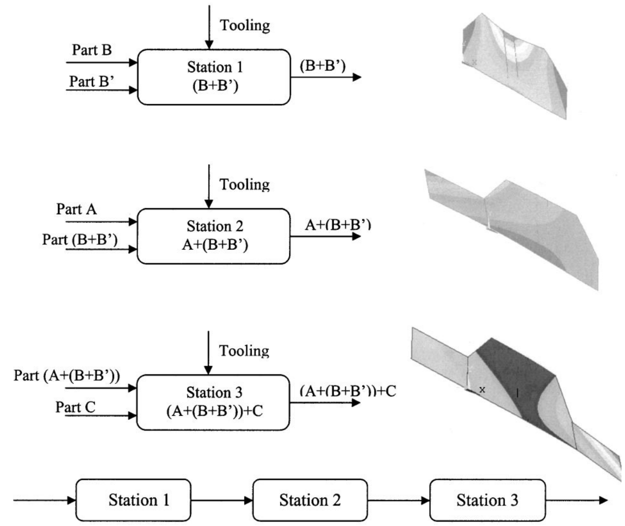

Fig. 9 Multi-station assembly example: right side structure (FEM models)

ditions like the welding locating points, the part locating points, and the measurement points. Figure 8 shows the locations of welding and locating points. In addition, process information is also required. This process information includes the assembly sequence, welding relations and the applied fixture associated to each station. Figure 9 shows the assembly sequence. All this information is share between the FEM and the variation propagation model.

Therefore, three different FEM models were defined for each of the three stations. The station and the mesh FEM models are showed in Fig. 9. The method of influence coefficients is applied for each station to obtain the deformation matrix **P** and the sensitivity matrix **S**. Relocation matrix **M** is defined using homogeneous transformation and the locators' positions. Finally, using the input variables for the model, the initial parts mean deviation and variance, and the tooling mean deviation and variance at each station, it is possible to run a Monte Carlo simulation. The simulation results are the mean deviation and standard deviation for the key measurement points expected of the final assembly.

Figure 10 shows results from the Monte Carlo simulation. A simulation was conducted for two cases:

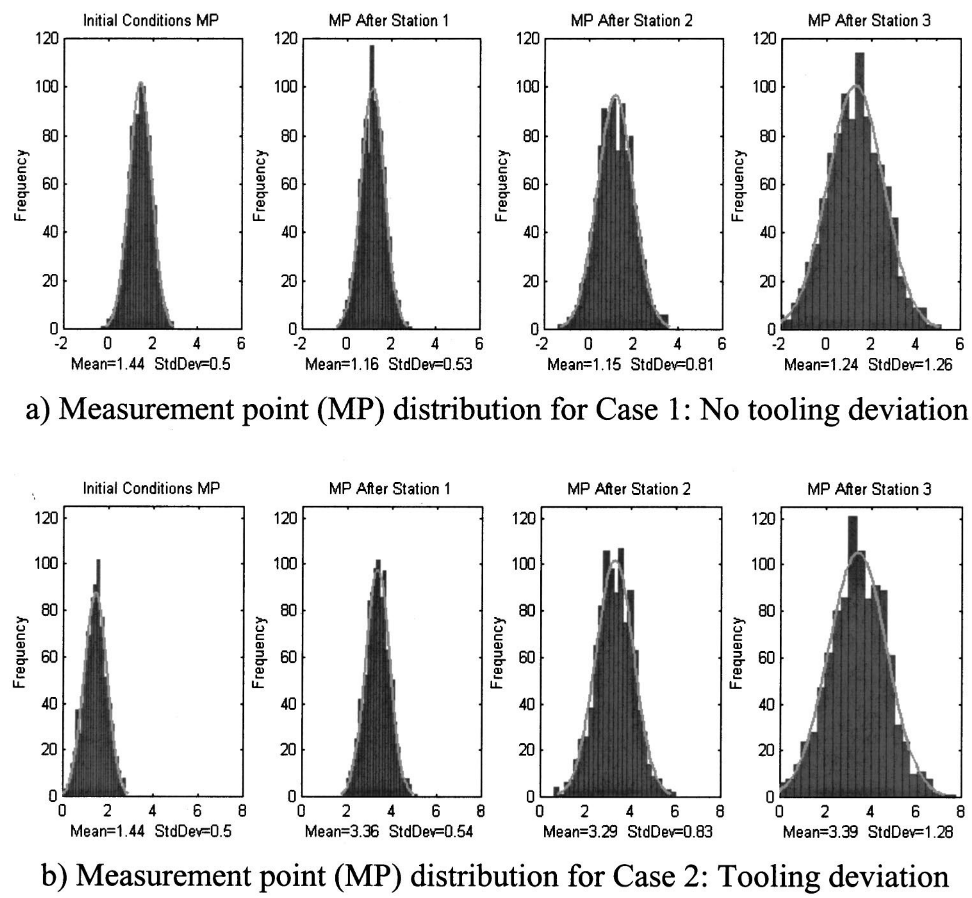

Fig. 10 Simulation results for the side structure

Table 2 State vector definition and initial conditions

X	WP A1	WP A2	LP A1	LP A2	LP A3	WP B1	WP B2	WP B3	WP B4	WP B5	WP B6	LP B1	LP B2	LP B3	MP B1
X(0)	0	0	0	0	0	0.29	−0.25	−0.32	−0.23	1	0	0	0	0	1.43
Std(0)	0.25	0.25	0.25	0.25	0.25	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0	0	0.5

X	WP B′1	WP B′2	LP B′1	LP B′2	LP B′3	WP C1	WP C2	LP C1	LP C2	LP C3
X(0)	0	0	0	0	0	1	0	0	0	0
Std(0)	0	0	0	0	0	0	0	0	0	0

- Case 1: Part deviation and variance were considered for parts A, B and C. and no fixture or tooling error were included. The state vector at station 0 (initial conditions) is shown in Table 2.
- Case 2: In addition to the previous part errors, locating fixture deviation and tooling deviation were included for each station. This case assumes that the tooling only have a mean deviation with no variation. The values for the vectors $\mathbf{U_g}$ and $\mathbf{U_t}$ at each station are presented in Table 3.

For both cases, the deviation distribution of one measurement point is plotted in Fig. 10 after each of the three stations. The selected measurement point is shown in Fig. 8.

3.2 Discussion. *Case 1:* The simulation results show that for serial assembly process (station 2 and station 3) the mean deviation and standard deviation of the measurement part increase. This is the effect of the reduction in the stiffness of the assembly and the relocation process. On the other hand, for parallel assembly process (station 1) the mean deviation and variation decrease, while the assembly stiffness increases. The above analysis shows that the additive theorem of variance accumulation widely assumed in variation simulation method is not valid for assembly process with compliant parts. These results agree with the work presented by Hu [2].

Case 2: Tooling variation can significantly affects the overall variation of the final assembly and intermediate subassemblies. If tooling variation is large enough, it may be not possible to state a difference between a serial and parallel assembly processes (additive and non-additive variance accumulation), the output mean deviation and variation will increase or decrease depending of the tooling impact in each station. In this example, the standard deviations for both cases at each station do not change since case 2 assumes that the tooling only have a mean deviation with no variation. However, if there is some tooling variance the results will be different. Therefore, it seems important during a new assembly process design to identify maximum tooling level variation in such a level that the non-additiveness of variation can be sustained. The non-additiveness of variance allows to design the assembly system in such way that the final variation of the product can be smaller that the variation of intermediate subassemblies and parts. In summary, the assembly process sequence evaluation depends not only on the assembly configuration but also on the input conditions such as: fixture and tooling variation. Therefore, the assembly process realization should consider all these factors during design stage to have optimally performing production system.

4 Conclusions

A new methodology is proposed for variation propagation analysis in multi-station compliant sheet metal assembly lines. The model presented uses a state space representation, where the state vector corresponds to the parts deviation. The method of influence coefficients presented by Liu and Hu [6], which was validated by experimentation, was applied to a multi-station system. Using this method and homogeneous transformation, it is possible to obtain the re-location matrix **M**, the deformation matrix **P** and the sensitivity matrix **S** for each station. In addition, the model considered the influence of part variation, fixture variation and welding gun variation in the final assembly variation. From the results in the case study, it can be concluded that the nonadditive characteristic of parallel assemblies may not be valid for certain levels of tooling and fixture variation. In other words, assembly variation can increase during parallel assembly. The selection of the optimal assembly sequence depends not only on of the assembly configuration but also on the input conditions such as: fixture and tooling variation.

Some general limitations of the proposed approach are simulation cost, data structure and continuity. For example, if the number of stations and parts increase, the number of FE models, simulations and simulation complexity also increases. Moreover, increasing the number of measurement points, locating points and welding points will result in an increase in the dimensions of the state vector, state matrix and input matrix. Finally, while the

Table 3 Vectors U_g and U_t at each station

	WP A1	WP A2	LP A1	LP A2	LP A3	WP B1	WP B2	WP B3	WP B4	WP B5	WP B6	LP B1	LP B2	LP B3	MP B1
Ug(1)	0	0	0	0	0	0	0	0	0	1	−1	0	0	0	0
Ug(2)	2	3	0	0	0	2	3	0	0	0	0	0	0	0	0
Ug(3)	0	0	0	0	0	0	0	−1	−1	0	0	0	0	0	0
Ut(1)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Ut(2)	0	0	0	0	0	0	0	0	0	0	0	0	−1	−1	0
Ut(3)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

	WP B′1	WP B′2	LP B′1	LP B′2	LP B′3	WP C1	WP C2	LP C1	LP C2	LP C3
Ug(1)	1	−1	0	0	0	0	0	0	0	0
Ug(2)	0	0	0	0	0	0	0	0	0	0
Ug(3)	0	0	0	0	0	−1	−1	0	0	0
Ut(1)	0	0	2	0	0	0	0	0	0	0
Ut(2)	0	0	0	0	0	0	0	0	0	0
Ut(3)	0	0	0	0	0	0	0	−1	−1	0

model is linear for different variations, it is discontinuous for other variables such as fixture position, given by the mesh, and the station index. Consequently, for a different welding scheme, a whole new model must be developed. New matrices **P** and **S** must be generated. This limitation is especially important when applying traditional optimization methods.

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Nomenclature

$\mathbf{A}(k)$ = state transition matrix ($n\times n$ matrix).
$\mathbf{B}(k)$ = input matrix ($n\times r$ matrix).
$\mathbf{C}(k)$ = output matrix ($m\times n$ matrix).
$\mathbf{D}(k)$ = direct transmission matrix ($m\times r$ matrix).
$\mathbf{F}_u$ = fixture force over the unwelded parts.
$\mathbf{F}_w$ = springback force over the welded assembly.
$\mathbf{I}$ = identity matrix ($n\times n$ matrix).
$\mathbf{K}$ = stiffness matrix.
$\mathbf{M}(k)$ = re-orientation matrix at station k ($n\times n$ matrix).
$\mathbf{P}(k)$ = deformation matrix before assembly at station k ($n\times n$ matrix).
$\mathbf{S}(k)$ = sensitivity matrix at station k ($n\times n$ matrix).
$\mathbf{U}(k)$ = input vector (n-vector) can include tooling deviation or welding gun deviations.
$\mathbf{U}_g(k)$ = welding gun deviation vector.
$\mathbf{U}_{t_{3-2-1}}$ = locating fixture deviation vector.
$\mathbf{U}_{t_{(N-3)}}$ = holding fixture deviation vector.
$\mathbf{V}_u$ = part deviation for the unwelded part.
$\mathbf{V}_w$ = springback deviation for the welded assembly.
$\mathbf{W}(k)$ = noise vector (n-vector).
$\mathbf{X}(k)$ = state vector (n-vector) represents the dimensional deviation of specific points in a global coordinate system.
$\mathbf{Y}(k)$ = output vector (m-vector) for interested measurement assembly points. It could be a sub-set of the state space vector.
MLP = measurement locating points.
PLP = part locator points.
WLP = welding locating points.
k = station index.
m = number of measurement points for the observation vector.
n = number of points in the state vector.

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