Compliant Assembly Variation Analysis Using Component Geometric Covariance

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Dimensional variation is one of the most critical issues in the design of assembled products. This is especially true for the assembly of compliant parts since clamping and joining during assembly may introduce additional variation due to part deformation and springback. This paper discusses the effect of geometric covariance in the calculation of assembly variation of compliant parts. A new method is proposed for predicting compliant assembly variation using the component geometric covariance. It combines the use of principal component analysis (PCA) and finite element analysis in estimating the effect of part/component variation on assembly variation. PCA is used to extract deformation patterns from production data, decomposing the component covariance into the individual contributions of these deformation patterns. Finite element analysis is used to determine the effect of each deformation pattern over the assembly variation. The proposed methodology can significantly reduce the computational effort required in variation analysis of compliant assemblies. A case study is presented to illustrate the methodology.
[DOI: 10.1115/1.1644553]

1 Introduction

Dimensional integrity is an important aspect of product quality in many manufactured consumer goods. Problems with dimensional integrity may adversely affect the final product functionality and the process performance. For example, large dimensional variation in an automotive body assembly may cause product problems in vehicles such as water leakage and wind noise, as well as process difficulties such as fitting problems in subsequent assembly operations.

Dimensional variation is inherent to any manufacturing process. Therefore, it is important to model and predict how it propagates through the processes. Fast and accurate evaluation of inherent process variation can be critical in determining the final dimensional variation of an assembled product and in selecting robust product/process designs. Several models have been proposed in the past to predict how variation propagates during assembly. Initial approaches were focused on rigid part assembly using either the Root Sum Squares (RSS) method or Monte Carlo Simulation. Detailed review and discussion can be found in Lee and Woo [1] and Chase and Parkinson [2]. Recently, multi-level variation propagation models have also been developed. Mantripragada and Whitney [3] proposed a state transition model to predict the variation propagation in multi-stage assembly systems. Ding et al. [4] presented a state space model for dimensional control in sheet metal assembly assuming rigid parts. For compliant assembly, Liu and Hu [5] proposed a compliant assembly model to analyze the effect of deformation and springback on assembly variation by applying linear mechanics and statistics. Using finite element methods (FEM), they constructed a sensitivity matrix to establish a linear relationship between the incoming part deviation and the output assembly deviation. They noted that the variation among the various nodes on a part may not be independent. As a result, the covariance of sources of variation must be included in the calculation of the assembly variation. Camelio et al. [6] extended Liu and Hu's approach to multi-station systems using state space representation by incorporating sources of variation from parts, tooling and their interactions.

The objective of this paper is to provide a new method of variation analysis for compliant assembly using the geometric covariance of the assembly components. It combines Principal Component Analysis (PCA) with finite element methods in estimating the effect of components variation on assembly dimensions. PCA is applied to extract deformation patterns from production data by decomposing the component covariance in the individual contribution of each deformation pattern. Finite element methods are used to determine the effect of each deformation pattern on the assembly variation. The proposed methodology is computationally more efficient than existing methods.

The remainder of this paper is organized as follows. Section 2 presents the background of the main concepts used in this paper. These concepts are compliant assembly variation analysis, geometric covariance and principal component analysis. Section 3 presents the new methodology to calculate assembly variation using deformation patterns. In Section 4, the proposed methodology is illustrated by an example. Finally, Section 5 draws the conclusions.

2 Background

This section presents the main concepts required to understand the proposed methodology. First, compliant assembly variation analysis is discussed and the concept of the sensitivity matrix is presented. Second, the idea of geometric covariance is presented. Finally, PCA is shown as a multivariate statistical tool to extract deformation patterns from measurement data.

2.1 Compliant Assembly Variation. Different variation analysis models are used to predict the effect of component variation on the assembly variation. In general, assembly variation is estimated as a function of the components geometry, process layout and the contribution of various sources of variation. Three sources of variation are identified in compliant sheet metal assembly: part or component variation, fixture variation and welding gun variation. Part variation includes the mean deviation, μ, and the variance of the deviation, σ^2 at key measurement locations. A deviation is the difference between the actual part dimension and the nominal dimension at a specific point in a given direction. In this paper, part deviation is denoted as a vector $\mathbf{V} \in \mathbf{R}^{n\times 1}$, in which the elements correspond to deviations at each key measurement locations.

Contributed by the Manufacturing Engineering Division for publication in the JOURNAL OF MANUFACTURING SCIENCE AND ENGINEERING. Manuscript received June 2003. Associate Editor: E. DeMeter.

Using the method of influence coefficients presented by Liu and Hu [5], it is possible to predict the impact of the part deviation ($\mathbf{V_u}$) on the assembly deviation ($\mathbf{V_a}$). This method is used to obtain the sensitivity matrix ($\mathbf{S}$) for a sheet metal assembly, where the elements of the sensitivity matrix, s_{ij}, measure the sensitivity of the assembly at node i to the incoming part deviation at node j. Therefore, the assembly deviation ($\mathbf{V_a}$) can be calculated using Eq. (1). By definition $\mathbf{V_a}$ is the assembly deviation vector, where the column elements represent the assembly deviation at the key measurement points. $\mathbf{V_u}$ is the component deviation vector, where the elements represent the component deviation at the welding nodes.

$$\mathbf{V_a}=\begin{bmatrix} v_{a_1} \\ v_{a_2} \\ \vdots \\ v_{a_m} \end{bmatrix}=\sum_{j=1}^{n}\begin{bmatrix} s_{1j} \\ s_{2j} \\ \vdots \\ s_{mj} \end{bmatrix}\cdot v_{u_j}$$

$$=\begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1n} \\ s_{21} & s_{22} & \cdots & s_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ s_{m1} & s_{m2} & \cdots & s_{mn} \end{bmatrix}\cdot\begin{bmatrix} v_{u_1} \\ v_{u_2} \\ \vdots \\ v_{u_n} \end{bmatrix} \tag{1}$$

$$\mathbf{V_a}=\mathbf{S}\cdot\mathbf{V_u}$$

Given that the input part deviations are random variables, the assembly quality can be described using the mean deviations and the variance of the assembly key characteristic points. Applying the expectation operator over Eq. (1), we have

$$E(\mathbf{V_a})=E(\mathbf{S}\cdot\mathbf{V_u}) \tag{2}$$

$$\boldsymbol{\mu}_\mathbf{a}=\mathbf{S}\cdot\boldsymbol{\mu}_\mathbf{u}$$

where $\boldsymbol{\mu}_\mathbf{a}$ and $\boldsymbol{\mu}_\mathbf{u}$ are the mean deviation vectors for the key points on the assembly and components, respectively.

In addition, the assembly covariance matrix can be calculated using the covariance definition:

$$\begin{aligned}\mathrm{cov}(\mathbf{V_a})&=E[(\mathbf{V_a}-\boldsymbol{\mu}_\mathbf{a})\cdot(\mathbf{V_a}-\boldsymbol{\mu}_\mathbf{a})^\mathbf{T}]\\&=E[\mathbf{S}\cdot(\mathbf{V_u}-\boldsymbol{\mu}_\mathbf{u})\cdot(\mathbf{V_u}-\boldsymbol{\mu}_\mathbf{u})^\mathbf{T}\cdot\mathbf{S}^\mathbf{T}]\\&=\mathbf{S}\cdot E[(\mathbf{V_u}-\boldsymbol{\mu}_\mathbf{u})\cdot(\mathbf{V_u}-\boldsymbol{\mu}_\mathbf{u})^\mathbf{T}]\cdot\mathbf{S}^\mathbf{T}\\&=\mathbf{S}\cdot\mathrm{cov}(\mathbf{V_u})\cdot\mathbf{S}^\mathbf{T}\end{aligned}$$

$$\boldsymbol{\Sigma}_\mathbf{a}=\mathbf{S}\cdot\boldsymbol{\Sigma}_\mathbf{u}\cdot\mathbf{S}^\mathbf{T} \tag{3}$$

$$\mathrm{cov}(\mathbf{V_u})=\boldsymbol{\Sigma}_\mathbf{u}=\begin{pmatrix} \sigma_{11}^2 & & & \cdots & \\ & \ddots & & & \vdots \\ & & \sigma_{ii}^2 & & \\ \sigma_{ij}^2 & & & \ddots & \\ & & & & \sigma_{nn}^2 \end{pmatrix}$$

where $\boldsymbol{\Sigma}_\mathbf{a}$ and $\boldsymbol{\Sigma}_\mathbf{u}$ represent the covariance matrix for the assembly and components, respectively.

The mean and covariance matrices for the input components must be obtained from statistical analysis of the measurements of production parts in the assembly line. In general, the production data is obtained using coordinate measurement machines (CMM) or optical coordinate measurement machines (OCMM). Equations (2) and (3) show that to estimate the assembly variation (mean and covariance) it is necessary to know beforehand the mean and covariance matrix of the components.

Under the assumption of independent sources of variation, the assembly variation could be calculated as:

$$\{\sigma_i^2\}_a=\sum_j S_{ij}^2\cdot\{\sigma_{jj}^2\}_u \tag{4}$$

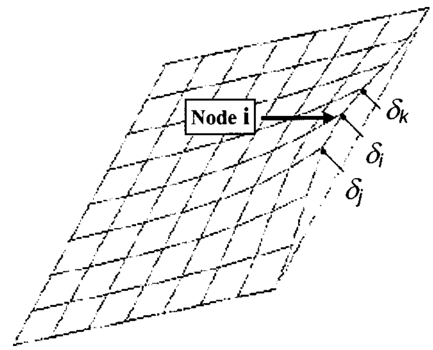


Fig. 1 Geometric covariance effect

However, a simple analysis shows that the assumption of independency is usually not adequate. In Eq. (4), assuming independent random variables, the assembly variation will increase as the number of welding points (sources of variation) increases. This phenomenon exists because as the number of sources increases, the welding points are getting closer to each other, and the independence assumption is forcing adjacent points to vary in different directions. Intuitively, close points on the same surface cannot vary independently. Moreover, intuition shows that if the number of welds increases, the assembly variation should decrease because the stiffness of the assembly increases with the number of welds (more constraints) and the springback will be smaller. Therefore, independency becomes questionable. To correctly estimate the assembly covariance matrix, it is required to know or estimate the components covariance matrix.

2.2 Geometric Covariance of Compliant Parts. As mentioned in the previous section, the physical relation among the neighboring points on the same surface must be considered. Intuitively, if a part is deformed by applying a displacement or force at a specific point, other points in the vicinity will follow the displacement of the point being deformed. As in the example shown in Fig. 1, if node i is displaced δ_i, nodes j and k in its vicinity will move δ_j and δ_k respectively. The amount of deformation on nodes j and k will depend on the geometric relationship between the nodes and the stiffness of the part.

The dependence in deviation among the adjacent points is called geometric covariance by Merkley [7]. Obviously, geometric covariance arises from surface continuity. This property is especially important during the assembly of compliant sheet metal parts. Merkley showed that geometric covariance clearly impacts the assembly variation of sheet metal assemblies as was shown by Eq. (3).

2.3 Principal Component Analysis of the Covariance Matrix. From the discussions in Section 2.1, it was shown that to estimate the assembly variation in compliant sheet metal assembly it is necessary to know the covariance matrix of the components to be assembled. In addition, the effect of geometric covariance in sheet metal parts implies that different sources of variation over the same surface are highly correlated. A new principle will be established in this section to take advantage of the geometric covariance effect and the covariance matrix. It will be shown that, in the presence of correlated sources of variation and with knowledge of the covariance matrix, it can be computationally more efficient to decompose the covariance matrix into different variation patterns. PCA will be applied to obtain the required decomposition of the covariance matrix.

PCA is a multivariate statistical method to reduce the dimensionality of highly correlated data via orthogonal projection into a space defined by a few significant eigenvectors [8,9]. Hu and Wu [10] were the first to use PCA to extract variation patterns from the correlation matrix in auto body assembly. PCA transforms a

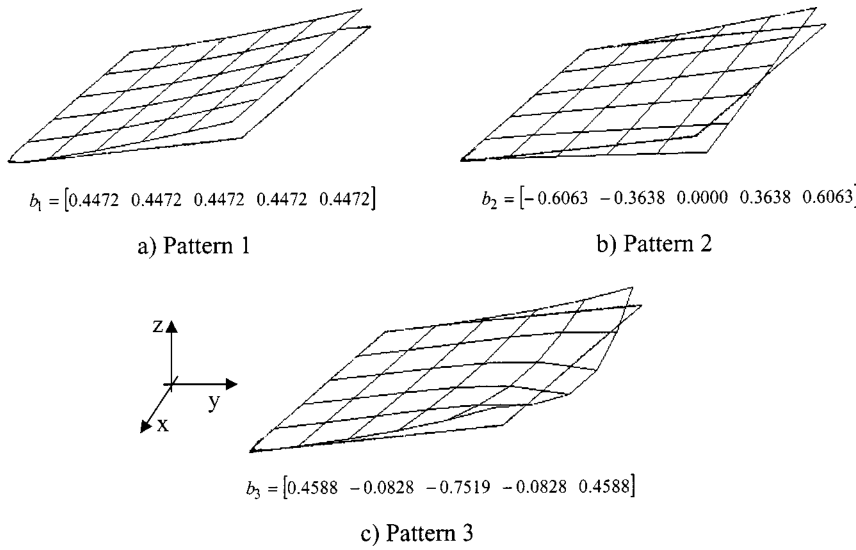


Fig. 2 Example deformation patterns

set of n dependent variables into a set of uncorrelated variables. If we consider the source of variation as the correlated variable X, using a linear transformation, we can transform our data into a set of uncorrelated data, Z. Then, the part variance can be decomposed into the individual contributions of these uncorrelated variables,

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{bmatrix} \cdot \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix} \tag{5}$$

where the vector $b_i = [b_{1i}\, b_{2i} \cdots b_{ni}]^T$ is the eigenvector i obtained from the principal component analysis applied to the covariance matrix $\mathbf{\Sigma_u}$. Then, each eigenvector ($\mathbf{b_i}$) of the covariance matrix will represent one variation pattern or mode of deformation of the assembly components, and its respective eigenvalue (λ_i) will correspond to the variance of that pattern. Rewriting Eq. (5) in a matrix form and calculating the covariance, we have:

$$\begin{aligned} \mathbf{X} &= \mathbf{B} \cdot \mathbf{Z} \\ \mathrm{cov}(\mathbf{X}) = \mathbf{\Sigma_u} &= E[(\mathbf{B} \cdot \mathbf{Z}) \cdot (\mathbf{B} \cdot \mathbf{Z})^T] \\ &= \mathbf{B} \cdot \mathbf{E}[\mathbf{Z} \cdot \mathbf{Z^T}] \cdot \mathbf{B}^T \\ &= \mathbf{B} \cdot \mathrm{cov}(\mathbf{Z}) \cdot \mathbf{B}^T \end{aligned} \tag{6}$$

Since $\mathbf{B}$ is the eigenvector matrix of $\mathbf{\Sigma_u}$, the covariance of $\mathbf{X}$ will be the covariance matrix $\mathbf{\Sigma_u}$ of the input data. In addition, from the definition of PCA, $\mathbf{Z}$ is a set of independent variables whose variances are the eigenvalues λ_i's. Then, Eq. (6) can be rewritten as:

$$\mathbf{\Sigma_u} = \mathbf{B} \cdot \Lambda \cdot \mathbf{B}^T \tag{7}$$

where,

$$\Lambda = \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix}$$

Physically, Eq. (7) means that the covariance matrix from the production data can be decomposed into different deformation patterns, where each pattern is mathematically represented by an eigenvector. The advantage of this decomposition in compliant assembly variation analysis will be shown in the next section.

Another useful property of PCA is that the different patterns are ordered so that the first eigenvector represents the variation pattern with the largest variance; the second eigenvector represents the pattern with the second largest variance, and so on. This property is extremely useful for variation reduction since usually only a small number of patterns will contribute significantly to the assembly variation. Therefore, PCA can also be applied as a valuable data reduction tool where the problem can be simplified by removing the deformation patterns with negligible variance.

Figure 2 shows some examples of deformation patterns for a sheet metal part that is fully constrained on one of its edges. Figures 2(a) and 2(c) show a bending pattern along the x and y axes, respectively. Figure 2(b) shows a twisting pattern. The deviation vector of each part is defined as the deviation from the nominal shape at equidistant points along the free edge. The corresponding deviation vectors (eigenvectors) are presented as the vector $\mathbf{b}$ in Fig. 2.

3 Variation Analysis Using Geometric Covariance

The key concept behind variation analysis for compliant sheet metal assembly is to establish a linear relationship between the deviations of incoming parts and the deviation of the assembly. Using the method of influence coefficients [5], the assembly variation can be calculated by Eq. (1). Calculating the covariance of the assembly variation vector $\mathbf{V_a}$ (Eq. (3)) and combining it with the decomposition from the PCA (Eq. (7)), we have,

$$\mathbf{\Sigma_a} = \mathbf{S} \cdot \mathbf{\Sigma_u} \cdot \mathbf{S^T} = \mathbf{S} \cdot \mathbf{B} \cdot \Lambda \cdot \mathbf{B}^T \cdot \mathbf{S^T} = (\mathbf{S} \cdot \mathbf{B}) \cdot \Lambda \cdot (\mathbf{S} \cdot \mathbf{B})^{\mathbf{T}} \tag{8}$$

Further analysis shows that the assembly covariance matrix can be decomposed into the individual contribution of each eigenvector from the component covariance matrix. Writing matrix $\mathbf{B}$ in terms of the eigenvectors $\mathbf{b_i}$,

$$\begin{aligned} \mathbf{S} \cdot \mathbf{B} &= \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1n} \\ s_{21} & s_{22} & \cdots & s_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ s_{m1} & s_{m2} & \cdots & s_{mn} \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{bmatrix} \\ &= \mathbf{S} \cdot [\mathbf{b_1} \ \ \mathbf{b_2} \ \cdots \ \mathbf{b}_n] \\ \mathbf{\Sigma_a} &= \mathbf{S} \cdot \mathbf{b_1} \cdot \lambda_1 \cdot (\mathbf{S} \cdot \mathbf{b_1})^{\mathbf{T}} + \mathbf{S} \cdot \mathbf{b_2} \cdot \lambda_2 \cdot (\mathbf{S} \cdot \mathbf{b_2})^{\mathbf{T}} + \ \ldots \\ &\quad + \mathbf{S} \cdot \mathbf{b_n} \cdot \lambda_n \cdot (\mathbf{S} \cdot \mathbf{b_n})^{\mathbf{T}} \end{aligned} \tag{9}$$

Defining $d_{\mathbf{i}} = \mathbf{S} \cdot \mathbf{b_i}$, then

$$\Sigma_a = d_1 \cdot \lambda_1 \cdot d_1^T + d_2 \cdot \lambda_2 \cdot d_2^T + \ldots + d_n \cdot \lambda_n \cdot d_n^T$$

The vector $\mathbf{d_i}$ will be called variation vector i, and it can be seen as the contribution of the ith-eigenvector to the assembly variation. In other words, the variation vector, $\mathbf{d_i}$, is the assembly deviation result for the displacements given by the ith-deformation pattern on the component.

The assembly variation can be decomposed in terms of the individual contribution of each eigenvector. In this analysis, we assume that the component covariance matrix is stationary, i.e., the deformation patterns do not change over time. In addition, in sheet metal assembly just a small number of deformation patterns should have a significant contribution on the covariance matrix. Then, the number of variables used to represent the assembly variation can be reduced from the original number of input sources of variation to a smaller number of significant deformation patterns. The assembly variance can be rewritten as.

$$\Sigma_a = d_1 \cdot \lambda_1 \cdot d_1^T + d_2 \cdot \lambda_2 \cdot d_2^T + \ldots + d_p \cdot \lambda_p \cdot d_p^T \tag{10}$$

where p is the number of significant modes of deformation.

Based on the above decomposition approach, a new methodology for variation simulation of sheet metal assembly processes is presented. From Eq. (10), it can be seen that the assembly covariance matrix can be obtained knowing the variation vectors, $\mathbf{d_i}$'s, and the variance of the deformation patterns, λ_i's. Therefore, instead of using the method of influence coefficients to generate the sensitivity matrix, a new approach is developed.

From Liu and Hu [5], the method of influence coefficients can be used to calculate the component response to a unit force applied at each of the N sources of variation (using N finite element runs) and the corresponding assembly springback for each source of variation (another N steps of FEM). Now, considering the concept of variation vectors, it is only necessary to calculate the component and assembly response to each deformation vector (eigenvectors). This method needs only $2p$ finite element runs.

The following steps describe how to calculate the vectors, $\mathbf{d_i}$, $i=1,\ldots,p$, using finite element methods.

Step 1: Component Response: A displacement vector, given by the significant eigenvectors $(i=1,\ldots,p)$, is applied over the "unwelded" components. FEM is used to calculate the force vector required to produce those displacements on each source of variation. The forces are recorded in a column vector.

$$\begin{bmatrix} F_{1i} \\ F_{2i} \\ \vdots \\ F_{ni} \end{bmatrix}$$

The process is repeated for each of the p deformation patterns. Then, the total force matrix is:

$$F = \begin{bmatrix} F_{11} & F_{12} & \cdots & F_{1p} \\ F_{21} & F_{22} & \cdots & F_{2p} \\ \vdots & \vdots & \vdots & \vdots \\ F_{n1} & F_{n2} & \cdots & F_{np} \end{bmatrix}$$

Step 2: Assembly Response: Each force vector $[F_i]$ $(i=1,\ldots,p)$ is applied over the "welded" structure (assembly) to calculate the resulting assembly displacement vector. The resulting vector is the assembly response or springback to the specific eigenvector deformation and corresponds to a variation vector, $\mathbf{d_i}$. The process is repeated for each of the p deformation patterns to obtain each of the vectors $\mathbf{d_i}$, $i=1,\ldots,p$.

Figure 3 shows the flow chart used to calculate the assembly variation using the eigenvector and eigenvalues obtained from the covariance matrix of the incoming components data.

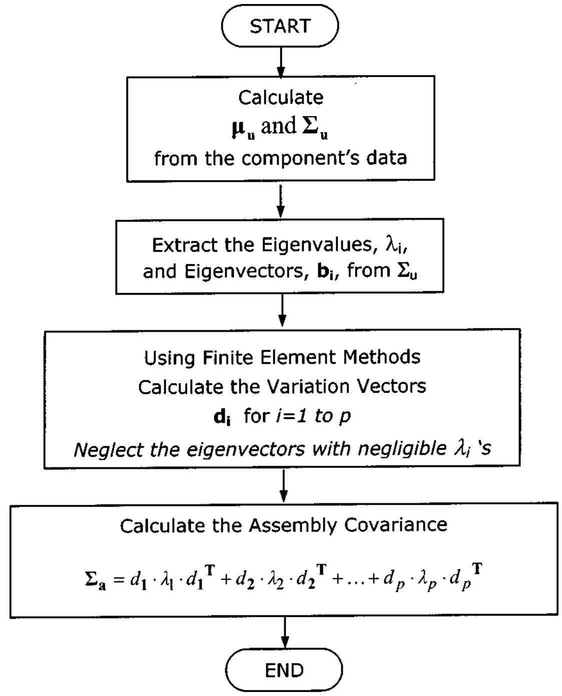


Fig. 3 Flow chart of the assembly variation methodology

4 Case Study

The proposed variation analysis method for compliant assembly is illustrated using the assembly of a safety crash bar to the inner door (Fig. 4). The door has approximate dimensions of 1000 mm×1 mm×500 mm. The safety bar has a thickness of 4 mm. The material of both parts is mild steel with Young modulus $E = 207{,}000\ \text{N/mm}^2$ and Poisson ratio $\nu=0.3$. A *3-2-1* locating scheme is used to locate each part. For the door, the locating scheme consists of a hole (constraint displacement in x and z directions) at node 626, a slot (constrained displacement in z-direction) at node 746, and 3 locating pads (constrained displacements in y-direction) at nodes $626\,(L_1)$, $746\,(L_2)$, and $489\,(L_3)$. For the safety bar, the locating scheme consists of a hole (constrained displacements in x and z directions) at node 5187, a slot (constrained displacement in z-direction) at node 5083, and 3 locating pads (constraint displacements in y-direction) at nodes $5187\,(L_1)$, $5083\,(L_2)$, and $5129\,(L_3)$. The parts are joined together using 4 welds that constrain nodes $5000/2974\,(W_1)$, $5002/2983\,(W_2)$, $5206/2719\,(W_3)$, and $5207/2939\,(W_4)$. Parts, fixtures and welding points are shown in Fig. 4.

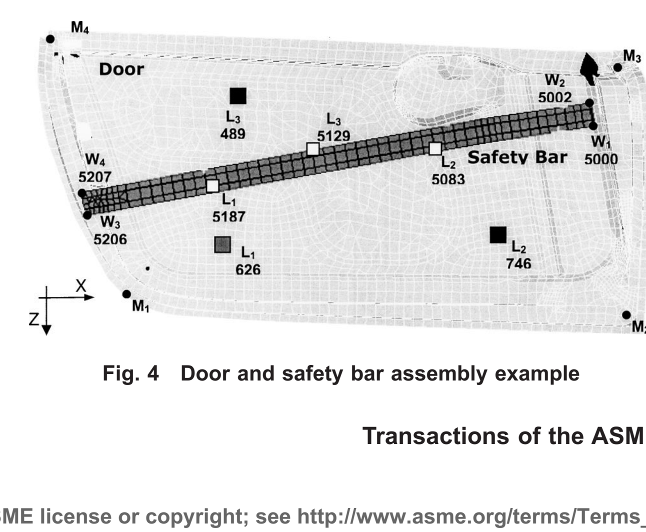
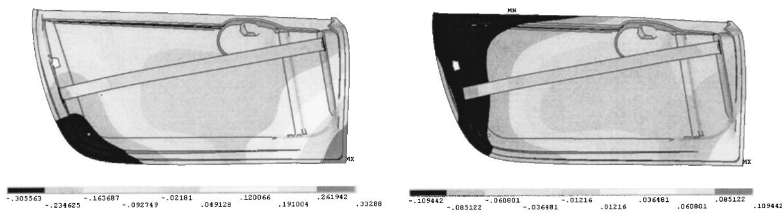


Fig. 4 Door and safety bar assembly example

Table 1 Principal component analysis summary

Deformation Patterns	Eigenvalue λ_i	% of Total Variance	Eigenvector $\mathbf{b_i}$
Pattern 1	2.0008	66.6	$\mathbf{b}_1=[0.7071\ 0\ -.07071\ 0]^{\mathbf{T}}$
Pattern 2	1.0001	33.3	$\mathbf{b}_2=[0.50\ 0.50\ 0.50\ 0.50]^{\mathbf{T}}$
Patterns 3–4	0.0000	0.0	$\mathbf{b}_3=[\ \ldots\ ]^{\mathbf{T}}$ *Neglected (No contribution)*



a) Pattern 1 ($\mathbf{d_1}$) b) Pattern 2 ($\mathbf{d_2}$)

Fig. 5 Assembly variation for each input variation pattern (door bar assembly)

Using finite element methods and the method of influence coefficients, the sensitivity matrix, **S**, can be obtained. The input variables are the sources of variation defined as the welding points, W_i, $i=1,\ldots,4$ on the safety bar (Fig. 4). The output variables are the measurement points, M_i, $i=1,\ldots,4$, on the door (Fig. 4).

$$\mathbf{S}=\begin{array}{c} \begin{array}{cccc} W_1 & W_2 & W_3 & W_4 \end{array} \\ \begin{bmatrix} -0.0246 & 0.0258 & 0.4073 & -0.2355 \\ 0.1833 & -0.1517 & -0.2878 & 0.1960 \\ 0.0148 & 0.0150 & 0.0656 & -0.0514 \\ 0.1654 & -0.1773 & 0.2234 & -0.0127 \end{bmatrix} \begin{array}{c} M_1 \\ M_2 \\ M_3 \\ M_4 \end{array} \end{array}$$

It is assumed that the input sources of variation from the safety bar (part deviations from the nominal before assembly) have the following mean, standard deviation and covariance matrix.

	W_1	W_2	W_3	W_4
Mean	−0.0078	−0.0024	0.0031	−0.0024
Standard Deviation	1.1138	0.4982	1.1171	0.4982

$$\mathbf{\Sigma_u}=\begin{bmatrix} 1.2405 & 0.2463 & -0.7478 & 0.2463 \\ 0.2463 & 0.2482 & 0.2500 & 0.2482 \\ -0.7478 & 0.2500 & 1.2478 & 0.2500 \\ 0.2463 & 0.2482 & 0.2500 & 0.2482 \end{bmatrix}$$

The covariance matrix has been simulated assuming that there are two deformation patterns in the data.

The door is assumed to be nominal. In other words, the assembly will be only affected by deformations (part variation) on the crash bar. Applying PCA, the input covariance matrix can be decomposed into the following pairs of eigenvectors, $\mathbf{b_i}$, and eigenvalues λ_i.

$$\mathbf{B}=\begin{bmatrix} 0.7089 & 0.5037 & 0.5000 & 0.0000 \\ 0.0018 & 0.5000 & -0.5000 & 0.7071 \\ -0.7053 & 0.4963 & -.5000 & 0.0000 \\ 0.0018 & 0.5000 & 0.5000 & -0.7071 \end{bmatrix}$$

$$\mathbf{\Lambda}=\begin{bmatrix} 2.0008 & & & \\ & 1.0001 & & \\ & & 0 & \\ & & & 0 \end{bmatrix}$$

From these results, it can be seen that there are two principal components that contribute for the total variation of the safety bar. Then, the system with initially 4 variables is reduced to a set of two independent variables. Moreover, 66% of the process variation can be explained by deformation pattern 1. Table 1 shows the eigenvectors and their respective contributions to the total input variation.

Knowing that the input variation can be explained by just two deformation patterns, the method of influence coefficients (sensitivity matrix) calculations can be reduced to obtain the assembly variation response to each of these patterns. Figure 5 shows the effect of each deformation pattern ($\mathbf{b_1},\mathbf{b_2}$) on the assembly. Applying the variation vectors methodology developed in this paper, it is possible to obtain the assembly variation vectors, $\mathbf{d_i}$, for each deformation pattern (Table 2). Figure 5 represents the variation vectors $\mathbf{d_1}$ and $\mathbf{d_2}$.

Using Eq. (10) and the assembly variation vectors, it is possible to calculate the assembly covariance matrix. Table 3 compares the covariance matrix calculation using the two methods, the method of influence coefficients and the method of assembly variation vectors presented in this paper. It can be seen that the results of both methods are similar. However, using the information from the geometric covariance from the input variables, it is possible to reduce the total number of computations required to calculate the assembly covariance matrix. Indeed, the number of FEM runs in the method of influence coefficients is proportional to the number of sources of variation. On the other hand, the number of FEM runs in the assembly variation vector method is proportional to the number of deformation patterns. Therefore, in cases where the data is highly correlated and there are few patterns but a large number of sources of variation, the computational effort can be

Table 2 Variation vectors

Deformation Patterns	Assembly Variation Vector
1	$\mathbf{d}_1=[-0.3051\ 0.3329\ -0.0363\ -0.0407]^{\mathbf{T}}$
2	$\mathbf{d}_2=[0.0864\ -0.0301\ 0.0217\ 0.0993]^{\mathbf{T}}$

Table 3 Comparison of the assembly covariance

	Assembly Covariance	
	Method of Influence Coefficients	Variation Vectors
	$\mathbf{\Sigma_a = S \cdot \Sigma_u \cdot S^T}$ $\Sigma_a = \begin{bmatrix} 0.1948 & -0.2070 & 0.0241 & 0.0338 \\ -0.2070 & 0.2238 & -0.0249 & -0.0304 \\ 0.0241 & -0.0249 & 0.0031 & 0.0052 \\ 0.0338 & -0.0304 & 0.0052 & 0.0133 \end{bmatrix}$	$\mathbf{\Sigma_a = D_1 \cdot \lambda_1 \cdot D_1^T + D_2 \cdot \lambda_2 \cdot D_2^T}$ $\Sigma_a = \begin{bmatrix} 0.1944 & -0.2065 & 0.0241 & 0.0335 \\ -0.2065 & 0.2234 & -0.0249 & -0.0302 \\ 0.0241 & -0.0249 & 0.0031 & 0.0051 \\ 0.0335 & -0.0302 & 0.0051 & 0.0132 \end{bmatrix}$
# FEM Runs	>8	4

substantially reduced by the new method. In general, the assumption that there is a reduced number of deformation patterns should be valid for real applications as is the case of the assembly of sheet metal parts, where surfaces tend to be smooth.

5 Conclusions

A new method for variation propagation analysis in compliant assembly has been presented using the covariance matrix of the components. The method replaces the method of influence coefficients [5] using variation vectors defined for each deformation pattern identified from the covariance of the components. The following observations were made:

• Geometric covariance plays an important role in the calculation of the assembly variation of compliant parts. In general, the sources of variation in sheet metal assembly are strongly correlated due to surface continuity. Assuming independent sources of variation produces an over estimation of the assembly variance.

• Principal component analysis (PCA) can be applied to extract deformation patterns from the covariance matrix. PCA may be used to reduce the number of variables in the problem, from n sources of variation to p significant deformation patterns.

• Assembly variation of compliant parts can be decomposed in terms of the individual contribution of the components deformation patterns. Neglecting deformation patterns with small variance, it is possible to reduce the number of independent sources of variation in the system.

• The variation vector methodology reduces the number of finite element computations required to calculate the assembly variation. The approach is based on the generation of variation vectors for the most significant deformation patterns.

Acknowledgment

The work presented in this paper was sponsored by the General Motors Collaborative Research Laboratory at the University of Michigan.

References

[1] Lee, W. J., and Woo, T. C., 1990, "Tolerances: Their Analysis and Synthesis," ASME J. Ind., **112**, pp. 113–121.

[2] Chase, K. W., and Parkinson, 1991, "A Survey of Research in the Application of Tolerance Analysis to the Design of Mechanical Assemblies," *Research in Engineering Design*, No. 3, pp. 23–37.

[3] Mantripragada, R., and Whitney, D. E., 1999, "Modeling and Controlling Variation Propagation in Mechanical Assemblies Using State Transition Models," IEEE Trans. Rob. Autom., **115**(1), pp. 124–140.

[4] Ding, Y., Ceglarek, D., and Shi, J., 2000, "Modeling and Diagnosis of Multi-Stage Manufacturing Process: Part I—State Space Model," *Japan-USA Symposium of Flexible Automation*, Ann Arbor, Michigan.

[5] Liu, S. C., and Hu, S. J., 1997, "Variation Simulation for Deformable Sheet Metal Assemblies Using Finite Element Methods," ASME J. Manuf. Sci. Eng., **119**, pp. 368–374.

[6] Camelio, J., Hu, S. J., and Ceglarek, D., 2003, "Modeling Variation Propagation of Multi-Station Assembly Systems With Compliant Parts," ASME J. Mech. Des. **125**(4).

[7] Merkley, K., 1998, "Tolerance Analysis of Compliant Assemblies," PhD Thesis, Brigham Young University.

[8] Johnson, R., and Wichern, D., 1998, *Applied Multivariate Statistical Analysis*, Prentice Hall, 4th Edition.

[9] Jackson, J. E., 1980, "Principal Component and Factor Analysis: Part I—Principal Components," J. Quality Technol., **12(4)**, pp. 201–213.

[10] Hu, S. J., and Wu, S. M., 1992, "Identifying Sources of Variation in Automobile Body Assembly Using Principal Component Analysis," Transactions of NAMRI/SME XX, pp. 311–316.