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Engineering Notes

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N-Impulse Orbit Transfer Using Genetic Algorithms

Ossama Abdelkhalik*
Embry–Riddle Aeronautical University, Daytona Beach, Florida 32114
and
Daniele Mortari†
Texas A&M University, College Station, Texas 77843

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Introduction

THE orbit transfer problems using impulsive thrusters have attracted researchers for a long time [1]. One of the objectives in these problems is to find the optimal fuel orbit transfer between two orbits, generally inclined eccentric orbits. The optimal two-impulse orbit transfer problem poses multiple local optima, and classical optimization methods find only local optimum solution. McCue [2] solved the problem of optimal two-impulse orbit transfer using a combination between numerical search and steepest descent optimization procedures. Jezewaski and Rozendall [3] developed an iterative method to calculate local minima solutions for the n-impulse fixed time rendezvous problems. Genetic algorithms (GAs) have been used in the literature to search for the global optimal orbit maneuver. Reichert [4] addressed the optimum two-impulse orbit transfer problem for coplanar orbits only. The accuracy obtained using this formulation is not good unless a narrow range, around the optimal value, for each design variable is known in advance [4]. Given narrow ranges for the design variables, the solution obtained using this formulation does not guarantee that the satellite will be inserted exactly into final orbit, but rather there is a small error unless the GA finds exactly the global optimal solution. Kim and Spencer [5] introduced a different formulation to the two-impulse orbit transfer problem by using six design variables for coplanar orbits. This formulation also does not guarantee the satellite is placed exactly in the final orbit.

In this note, a new formulation to the problem is introduced. This formulation is general for noncoplanar elliptical orbits. It can also implement any number of thrust impulses. For the case of two-impulse maneuver, this formulation requires only three design variables for any noncoplanar orbit transfer. The solution obtained by this formulation is guaranteed to insert the satellite in the final orbit exactly, even if the GAs did not converge to the global optimal solution. This formulation requires solving Lambert's problem to find the parameters of the transfer orbit for a given set of the three design variables. The next section describes the orbit maneuver algorithm. The two-impulse transfer is considered a special case and is presented separately. Validation to this formulation is performed by solving several case studies to which the optimal solution is known.

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*Visiting Assistant Professor, College of Engineering, 600 South Clyde Morris Boulevard, Room LB356; abdelkho@erau.edu.

†Associate Professor, Department of Aerospace Engineering, H.R. Bright Building, Room 611C; mortari@tamu.edu.

Problem Formulation

Two-Impulse Maneuver

Consider a satellite in an initial orbit defined by the five orbital elements a_I, e_I, i_I, ω_I, and Ω_I. The final orbit is defined by the five orbital elements: a_F e_F i_F ω_F, and Ω_F. Assume that the satellite is subject only to the Newtonian gravitational force. The true anomaly on the initial orbit at the time of satellite departure from the initial orbit is θ_I. The true anomaly on the final orbit at the time of satellite arrival to the final orbit is θ_F. The position and velocity of the satellite on the initial orbit at the time of departure are r_I and v_I, respectively. The position and velocity of the satellite on the final orbit at the time of arrival are r_F and v_F, respectively. The geometry for the orbit transfer is shown in Fig. 1.

The objective function that will be minimized is selected to be the total cost of the maneuver, which is the sum of the initial and final impulses

$$f = \sum \|\Delta v\| = \|v_I - v_{\mathrm{It}}\| + \|v_F - v_{\mathrm{Ft}}\| \tag{1}$$

The design variables are selected to be θ_I, θ_F, and the time of flight on the transfer orbit from the initial position to the final position, t_f. It is required to find the optimal values for the design variables that minimize the objective function. A genetic algorithm requires the evaluation of the objective function based on given values for the design variables. This can be done as follows [6]. The initial orbit is known, and for a given value of the design variable θ_I, the position of the satellite on the initial orbit is completely defined by the six orbital elements. From the six orbital elements, the position r_I, and the velocity v_I, of the spacecraft on the initial orbit at the first impulse location are computed [7]. Similarly, the final orbit is known, and for a given value of θ_F, r_F, and v_F of the spacecraft on the final orbit at the second impulse location are computed. For a given value for the design variable t_f, and the calculated values for r_I, r_F, we can solve the Lambert's orbital two point boundary value problem [8] to calculate the orbital elements of the transfer orbit. From the transfer orbit parameters, we can calculate the velocities on the transfer orbit at the initial and final positions, v_{It} and v_{Ft}, respectively. The required Δv, for the given set of design variables, is then calculated from Eq. (1).

In the case in which the transfer angle between the initial position and the final position is π or 2π, then the preceding design variables do not completely specify a solution. In this case, and if the initial and final orbits planes are coplanar, then the designer should select the transfer orbit plane as the same plane of the initial and final orbits. If the initial and final orbits are not coplanar, then additional variable(s) should be added to completely specify the transfer plane. This special case is not considered in this note.

After we get the solution of the GA, we can apply a steepest descent algorithm using the GA solution as an initial guess. This will guarantee the final solution is at a local optima.

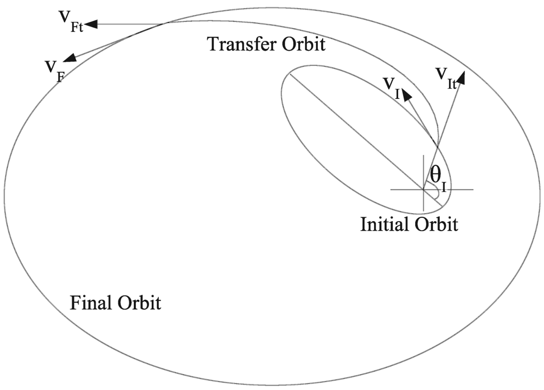

Fig. 1 Geometry for orbit transfer.

N-Impulse Maneuver

In an n-impulse orbit transfer, the number of transfer orbits is $n-1$ where n is the number of impulses. For the last two impulses, the design variables are selected exactly like the two-impulse case presented in the preceding section: the true anomaly on orbit $n-1$ at the $n-1$ impulse, the true anomaly on the final orbit at the last impulse n, and the time of flight on the last transfer orbit, t_f. For impulses before the last two, the design variables are selected to be the velocity impulse vector, and the true anomaly on the departing orbit for that impulse. Therefore, the total design variables are 1) all the velocity impulses vectors except the last two; 2) Δv_i for impulse i, where $i = 1, \cdots, n-2$; 3) θ_i, the true anomaly on orbit i for the ith impulse, where $i = 1, \cdots, n-1$;4) θ_F, the true anomaly on the final orbit for the final impulse; and 5) t_f, the time of flight on the last transfer orbit. The number of design variables is then $4 \times (n-2) + 3$. Consider, for example, the case of the three-impulse maneuver illustrated in Fig. 2. The design variables are the first impulse Δv_1, the true anomaly on the initial orbit for the first impulse θ_1, the true anomaly for the second impulse on orbit 2, θ_2, the true anomaly for the last impulse on the final orbit, θ_F, and the time of flight on orbit 3, t_f.

The objective function is the total cost of the maneuver, which is the sum of all thrust impulses

$$f = \sum_{1}^{n} \|\Delta v_i\| \tag{2}$$

For a given design point, Δv_i for $i = 1, \cdots, n-2$ are simply design variables and no calculations are needed to include them in the objective function. The last two impulses, however, are not design variables and are calculated in a way similar to that used in the two-impulse objective function evaluation. To evaluate the last two impulses, we need to calculate the orbit $n-1$. In a general n-impulse orbit transfer, this is done by starting from the initial orbit, and given Δv_1 and θ_1, we calculate r_1 and v_{1t}, and use them to evaluate the orbital elements of the transfer orbit 2. Given θ_2, we calculate r_2 and v_{2t}, and use them to evaluate the orbital elements of the transfer orbit 3. We continue until we evaluate the orbit elements for the orbit $n-1$. Finally, we use the orbit elements of orbit $n-1$, θ_{n-1}, θ_F, t_f, and the orbit elements of the final orbit $n+1$ to evaluate Δv_{n-1} and Δv_n by solving Lambert's problem, as done in the two-impulse transfer case.

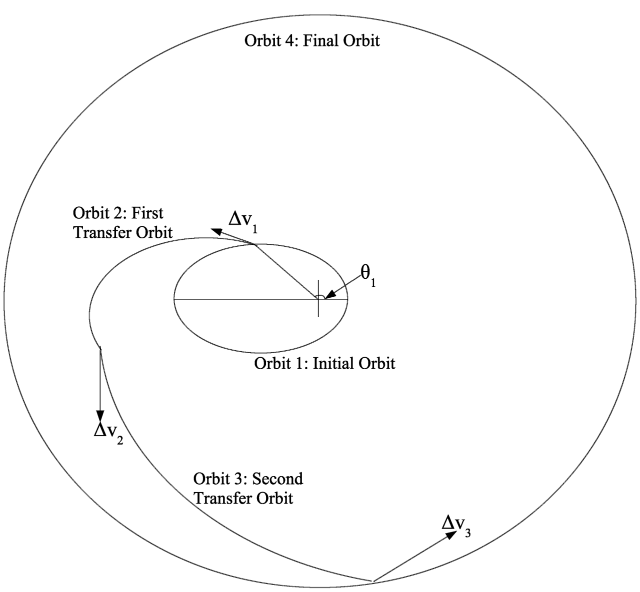

Fig. 2 Three-impulse orbit transfer geometry.

Results

For the purpose of validation, the developed algorithm is applied to known problems and results are compared with the known solutions. More case studies are available in [6].

Problem 1: The Hohmann Transfer

In the Hohmann transfer maneuver, both initial and final orbits are circular. The optimal solution to the Hohmann transfer problem is a two-impulse maneuver. It is characterized by the point of departure from the initial orbit, the point of arrival on the final orbit, and the center of the central body are all aligned. This can be written as $\theta_F = \theta_I + \pi$. As a case study, consider the transfer from a circular Mars orbit of radius 8000 km to a circular Mars orbit of radius 15,000 km. The optimal solution is the Hohmann transfer with a required total velocity change of 0.609 km/s. The time required for transfer is 5.08 h [9].

The solution provided by the GA is shown in Figs. 3 and 4. Figure 3 shows a plot for the transfer orbit. The aforementioned three points are almost aligned, which implies that this is the optimal solution. Figure 4 shows the number of iterations and the final values for θ_I, θ_F, and t_f, and their occurrences in the final generation. In Fig. 4, M_1 is θ_I and M_2 is θ_F. The required total velocity change is 0.60928 km/s. The error is 0.045%. The true anomaly at departure is 127.85 deg. The true anomaly at arrival is 306.73 deg. The difference between them is 178.88 deg, compared with the exact optimal value of 180 deg. The calculated transfer time is 5.157 h, compared with the exact optimal value of 5.08 h. The satellite is transferred exactly to the final orbit with no error. The slight drift from the optimal solution appears as a slightly higher transfer time and, consequently, a slightly higher total Δv. The method converged to the optimal solution after about five iterations.

Problem 2: Transfer from Parking Orbit to Geosynchronous Orbit

Consider the case of transferring a spacecraft from a circular Earth parking orbit of radius 6671.53 km and inclination of 28.5 deg, to a

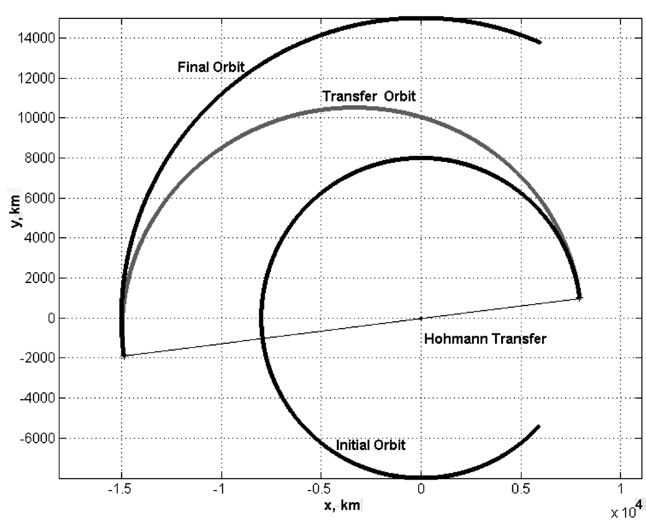

Fig. 3 Solution orbit of the Hohmann transfer problem using GA.

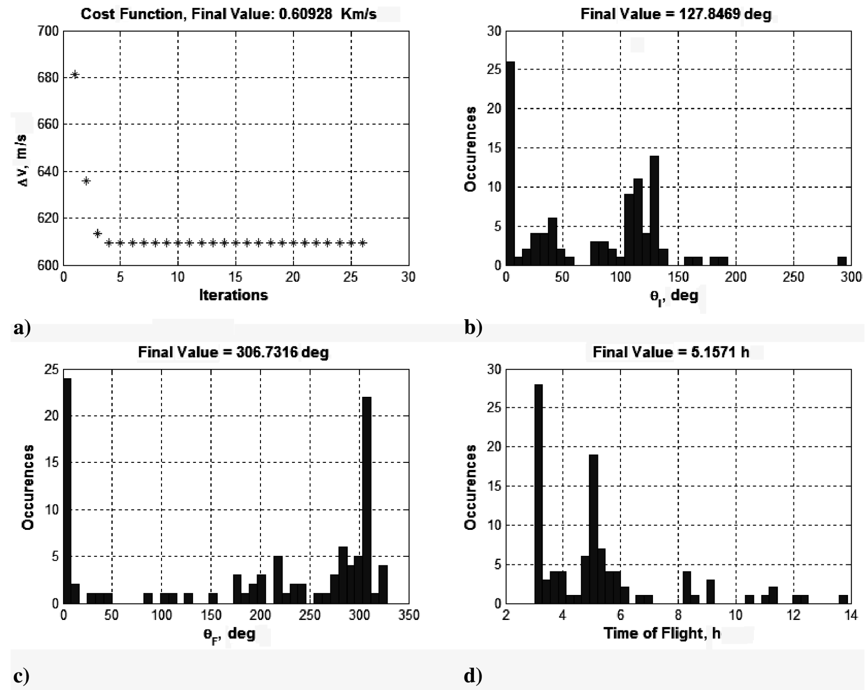

Fig. 4 The Hohmann transfer solution: a) cost function convergence vs generations, histogram in the last population of the design variables; b) departure true anomaly; c) arrival true anomaly; and d) time of flight.

Table 1 Minimum plane changes and $\|\Delta v\|$ for a low earth orbit to GEO transfer

Solution	Δi_I	Δi_F	$\|\Delta v\|$, km/s
1	3.305 deg	25.195 deg	4.05897
2	3.244 deg	25.256 deg	4.0590
3	3.3003 deg	25.2574 deg	4.0610

geostationary orbit (GEO) of radius 26,558.56 km and zero inclination. The optimal solution is to perform a small inclination change along with the orbit raising at the first impulse and then perform most of the orbit inclination along with the circularization in the second impulse [7].

The results are summarized in Table 1. The amount of inclination change performed at the initial impulse is Δi_I. The amount of inclination change performed at the final impulse is Δi_F. The total cost of the maneuver is Δv. Solution 1 is the optimal solution as presented by Vallado [7]. Solution 2 is the final solution as obtained by the developed tool after we find the local minima close to the GA solution. Solution 3 is the solution obtained directly from the GA. The final solution obtained by the developed GA tool is almost identical to the optimal solution.

Problem 3: Three-Impulse Orbit Transfer

To demonstrate the efficiency of the method in an n-impulse orbit transfer case, the case of the three-impulse transfer is considered. The problem solved here is the same problem addressed by Kim and Spencer [5]. The initial orbit is a circular orbit of radius 7000 km. The final orbit is also circular and its radius is 42,164 km. The two orbits are coplanar. The solution obtained by Kim and Spencer [5] has a

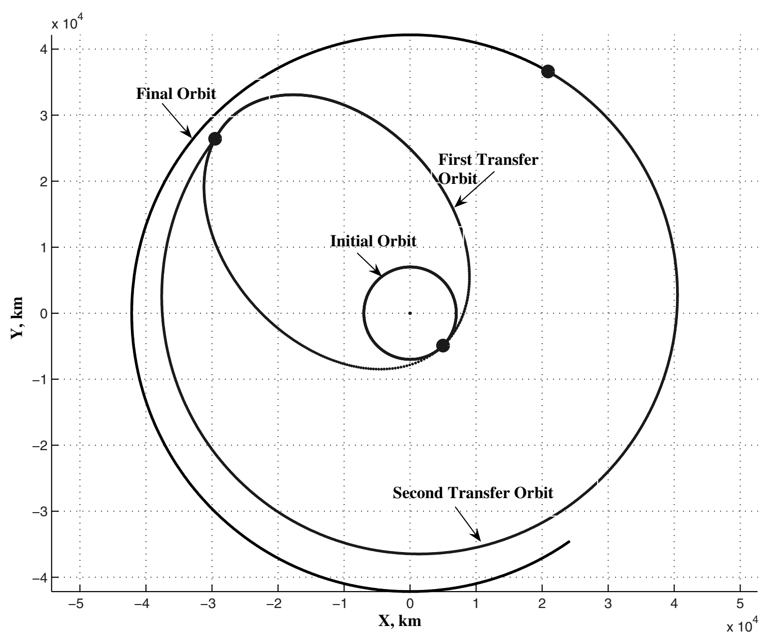

Fig. 5 Three-impulse orbit transfer solution.

Table 2 Three-impulse orbit transfer solution $\|\Delta v\|$

Burn	$\|\Delta v\|$, km/s	θ, deg
First	2.3303	315.6
Second	1.4196	186.1
Third	0.1210	60.4

total Δv of 4.549 km/s. The solution obtained using the developed method has a total Δv of 3.87 km/s. The solution is presented in Fig. 5 and the three velocity impulses are shown in Table 2.

Discussion

The formulation for the orbit transfer problem has the following advantages. First, it is guaranteed to insert the spacecraft in the final orbit exactly even if the transfer is not optimal. A nonoptimality in a solution will appear as more required fuel and longer maneuver time. The reason relies on the fact that for each member in the population, the Lambert's problem is solved. The solution of the Lambert's problem yields the exact orbit transfer from the member's initial position to its final position during its assigned time of flight. And so, the most fit member in the last generation, which is the solution provided by the GA, will exactly arrive in the final orbit with no error in position.

Second, the number of design variables is only three in the two-impulse transfers, even for the case of noncoplanar elliptical initial and final orbits. This is compared with six design variables in coplanar orbits and eight design variables in noncoplanar orbits in the formulation introduced by Kim and Spencer [5] and compared with three design variables in only coplanar orbits in the formulation introduced by Reichert [4]. Because of the lower number of design variables, the running time for this algorithm is relatively small. This small running time allows for more exploration for the design space. In implementing a GA, it is required to compromise between exploitation of the most fit members and exploration for the design space. Exploitation causes a fast convergence to a solution; however, some regions in the design space may not be investigated, and the optimal solution could be in these regions. Exploration is slower in finding the final solution but has more probability to hit the true optimal solution. Usually a tradeoff between exploitation and exploration is tailored for each problem. In the orbital maneuver problem, and because of the smaller number of variables and fast running time, it is possible to give more time for exploring new solutions and so increase the probability of hitting the true optimal solution.

Conclusions

The developed algorithm demonstrated fast convergence to almost global optimal solutions for all the examples presented in the note. The method is efficient and transfers the spacecraft exactly to the final orbit even if the solution is not exactly the global optimal. Only three design variables are needed for a general transfer between two noncoplanar conic orbits.

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B. Marchand
Associate Editor