Constitutive Modeling of Electrostrictive Polymers Using a Hyperelasticity-Based Approach

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The use of constitutive equations to describe the electromechanical behavior of electrostrictive materials began over 100 years ago. While these equations have been used to model a host of ceramic-based and polymer-based electroactive materials, a fully characterized model has not yet been developed to predict the response of transversely isotropic polymer electrostrictives. A constitutive model is developed within a thermodynamic and hyperelastic framework that incorporates the transversely isotropic material symmetry that is present in many polymer-based electrostrictives. The resulting constitutive model is characterized for three electrostrictive polymer systems using empirical data that are available in the literature. The model has a relatively simple functional form that is easily adaptable to other polymer electrostrictive material systems. [DOI: 10.1115/1.3173766]

Keywords: electroactive materials, electrostriction, MEMS, polyurethane elastomers

1 Introduction

Polymer-based electrostrictive materials became a focal point of research for applications that require large magnitudes of actuation and significant weight-savings [1–23]. These applications range from microelectromechanical systems (MEMS) to artificial muscles. The use of these materials in engineering applications requires accurate and flexible constitutive relations to relate loads, deformation, electric displacement, and applied electrical field.

In the late 19th century and the early part of the 20th century, several studies reported a relationship between the applied electric field, stress field, and strain field for cylindrical condensers [24–33]. In the middle portion of the 20th century, many scholars studied the broader applications of electrostrictive materials and established more specific definitions of electrostrictive behavior. In 1941, Stratton [34] defined electrostriction as simply the elastic deformation of a dielectric under the forces exerted by an electrostatic field. However, numerous authors [35–42] generally agreed that electrostrictive materials exhibit a constitutive behavior that can be described in a thermodynamic framework. More recently, many studies employed this thermodynamic framework for electrostrictive materials [43–56].

The objective of this study is to establish a hyperelastic framework for modeling polymer-based electrostrictive materials with transverse-isotropic material symmetry. The resulting constitutive model is characterized for three electrostrictive polymer systems using empirical data that are available in the literature. The thermodynamic framework of the modeling approach is established first followed by the characterization of material parameters.

2 Kinematics and Balance Laws

Consider a region of a material manifold $\mathfrak{R}$ embedded in a three-dimensional Euclidean space with its volume enclosed by the surface $\partial\mathfrak{R}$. A material point of the region in its reference state is located by its rectangular coordinate vector $\mathbf{X}$ at time $t=0$, whose components are taken with respect to the mutually perpendicular basis set $e=\{\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3\}$. For any time $t>0$, the region deforms to the spatial configuration $\mathfrak{R}_t$ with surface $\partial\mathfrak{R}_t$. The coordinate of the material point in the spatial configuration is given by the vector $\mathbf{x}$. The coordinates in the reference and spatial configurations are related by

$$\mathbf{x}=\chi(\mathbf{X},t) \tag{1}$$

The deformation gradient tensor is given by

$$F_{iK}=\frac{\partial x_i}{\partial X_K} \tag{2}$$

The right Cauchy–Green deformation tensor is defined as $\mathbf{C}=\mathbf{F}^T\mathbf{F}$, whose eigenvalues are the squares of the principal stretches associated with the deformation of Eq. (1). The region is subjected to an electric field vector denoted in $\mathfrak{R}$ as the Lagrangian electric field $\mathbf{E}$.

Over $\mathfrak{R}$, the mass balance is

$$\dot{\rho}=0 \tag{3}$$

where the superposed dot indicates a material derivative and ρ is the mass density of the material. The balance of linear momentum, assuming static conditions and no body forces, is

$$\text{div}\,\mathbf{FS}=\mathbf{o} \tag{4}$$

where div is the divergence operator with respect to the reference configuration, $\mathbf{S}$ is the second Piola–Kirchhoff stress tensor, and $\mathbf{o}$ is the null vector. The angular momentum balance of the system is simply the proof of the symmetry of the second Piola–Kirchhoff stress tensor.

$$\mathbf{S}=\mathbf{S}^T \tag{5}$$

These three balance principles can be easily established using standard techniques [57,58]. The energy balance in the reference configuration, which includes the energy of the electric field, is [39,59–61]

$$-\rho\dot{U}+(1/2)\mathbf{S}:\dot{\mathbf{C}}+\mathbf{E}\cdot\dot{\mathbf{D}}+\text{div}\,\mathbf{Q}+\rho h=0 \tag{6}$$

where U is the specific internal energy, $\mathbf{D}$ is the Lagrangian electric displacement vector, $\mathbf{Q}$ is the heat flux vector, and h is the volumetric thermal heat source. It has been shown that $\mathbf{E}$ and $\mathbf{D}$ are work conjugates for a deformable dielectric [62]. The second law of thermodynamics for the reference configuration is [59]

$$\rho\dot{\eta}-\frac{1}{\theta}\text{div}\,\mathbf{Q}+\left(\frac{1}{\theta^2}\right)\mathbf{Q}\cdot\text{grad}\,\theta-\frac{\rho h}{\theta}\geq 0 \tag{7}$$

where θ is the temperature and grad is the gradient function with respect to the reference configuration. The free energy of the systems is defined as [59]

$$\psi=U-\theta\eta-\left(\frac{1}{\rho}\right)\mathbf{E}\cdot\mathbf{D} \tag{8}$$

The Clausius–Duhem inequality is established by substitution of Eqs. (6) and (8) into Eq. (7)

$$-\rho(\dot{\theta}\eta+\dot{\psi})+(1/2)\mathbf{S}:\dot{\mathbf{C}}-\dot{\mathbf{E}}\cdot\mathbf{D}+\left(\frac{1}{\theta}\right)\mathbf{Q}\cdot\text{grad}\,\theta\geq 0 \tag{9}$$

Therefore, Eqs. (3)–(6) constitute 8 scalar field equations for 22 scalar quantities (ρ, $\mathbf{S}$, $\mathbf{x}$, $\mathbf{D}$, ψ, θ, $\mathbf{E}$, and η with h prescribed).

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Contributed by the Applied Mechanics Division of ASME for publication in the JOURNAL OF APPLIED MECHANICS. Manuscript received August 11, 2008; final manuscript received April 13, 2009; published online October 5, 2009. Review conducted by Zhigang Suo.

